Lineær algebra

Matriser er todimensjonale tabeller
bestående av m rader og n kolonner
og inneholdende reelle og/eller komplekse tall

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

Eks: En 3 x 4 matrise (3 rader og 4 kolonner)
inneholdende naturlige tall

$$A = \begin{bmatrix} 2 & 7 & 0 & 1 \\ 0 & 5 & 3 & 3 \\ 8 & 9 & 0 & 4 \end{bmatrix}$$

Lineær algebra

Addisjon av matriser utføres cellevis og krever at begge matrisene har samme dimensjon.

Eks:

$$A = \begin{bmatrix} 2 & 7 & 0 & 1 \\ 0 & 5 & 3 & 3 \\ 8 & 9 & 0 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 4 & 0 & 3 \\ 3 & 5 & 1 & 0 \end{bmatrix}$$

$$C = A + B \qquad c_{ij} = a_{ij} + b_{ij}$$

$$C = A + B = \begin{bmatrix} 2 & 7 & 0 & 1 \\ 0 & 5 & 3 & 3 \\ 8 & 9 & 0 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 4 & 0 & 3 \\ 3 & 5 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 7 & 0 & 3 \\ 2 & 9 & 3 & 6 \\ 11 & 14 & 1 & 4 \end{bmatrix}$$

Lineær algebra

Multiplikasjon av matriser utføres radvis mot kolonnevis og har følgende dimensjonskrav:

A m x n matrise

B p x q matrise

$\Rightarrow$ n = p C = AB n x q matrise

$$C = AB \qquad c_{ij} = \sum_k a_{ik} b_{kj}$$

Eks: $A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 4 & 5 \end{bmatrix}$ $\qquad B = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 4 & 0 & 3 \\ 3 & 5 & 1 & 0 \end{bmatrix}$

$$C = AB = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 4 & 5 \end{bmatrix} \begin{bmatrix} 2 & 7 & 0 & 1 \\ 0 & 5 & 3 & 3 \\ 8 & 9 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2\cdot 2 + 1\cdot 0 + 3\cdot 8 & 2\cdot 7 + 1\cdot 5 + 3\cdot 9 & 2\cdot 0 + 1\cdot 3 + 3\cdot 0 & 2\cdot 1 + 1\cdot 3 + 3\cdot 4 \\ 0\cdot 2 + 4\cdot 0 + 5\cdot 8 & 0\cdot 7 + 4\cdot 5 + 5\cdot 9 & 0\cdot 0 + 4\cdot 3 + 5\cdot 0 & 0\cdot 1 + 4\cdot 3 + 5\cdot 4 \end{bmatrix} = \begin{bmatrix} 28 & 46 & 3 & 17 \\ 40 & 65 & 12 & 32 \end{bmatrix}$$

Merk at matrismultiplikasjon ikke er en kommutativ operasjon.

Den transponerte matrisen A^T av en matrise A fremkommer ved å bytte rader og kolonner.

$$a^T_{ij} = a_{ji}$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \dots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \dots & a_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix} \qquad A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} & \dots & a_{m1} \\ a_{12} & a_{22} & a_{32} & \dots & a_{m2} \\ a_{13} & a_{23} & a_{33} & \dots & a_{m3} \\ \dots & \dots & \dots & \dots & \dots \\ a_{1n} & a_{2n} & a_{3n} & \dots & a_{mn} \end{bmatrix}$$

Lineær algebra

2 x 2 matrise	$A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $\det(A)=\lvert A\rvert=\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}=a_{11}a_{22}-a_{12}a_{21}$
3 x 3 matrise	$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{13} \\ a_{31} & a_{32} & a_{13} \end{bmatrix}$ $\det(A)=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{13} \\ a_{31} & a_{32} & a_{13} \end{vmatrix}=a_{11}\begin{vmatrix} a_{22} & a_{13} \\ a_{32} & a_{13} \end{vmatrix}-a_{12}\begin{vmatrix} a_{21} & a_{13} \\ a_{31} & a_{13} \end{vmatrix}+a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Lineær algebra

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} \qquad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

Lineær algebra

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{13} \\ b_3 & a_{32} & a_{13} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{13} \\ a_{31} & a_{32} & a_{13} \end{vmatrix}} \qquad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{13} \\ a_{31} & b_3 & a_{13} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{13} \\ a_{31} & a_{32} & a_{13} \end{vmatrix}} \qquad x_3 = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{13} \\ a_{31} & a_{32} & a_{13} \end{vmatrix}}$$

Lineær algebra

Med en invers matrise A^{-1} (hvis den finnes) til en gitt matrise A mener vi en matrise A^{-1} som multiplisert med matrisen A (både fra venstre og høyre) gir identitsmatrisen.

$$AA^{-1} = A^{-1}A = I = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Kun kvadratiske matriser kan ha en invers matrise.
Eksistens av invers matrise A^{-1} er ekvivalent med at $\det(A) \neq 0$.
Identitetsmatrisen er entydig.

Lineær algebra

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad A^{-1} = \frac{1}{|A|} \begin{bmatrix} a_{22} & -a_{12} \\ a_{21} & a_{11} \end{bmatrix}$$

Lineær algebra

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_1 \to \frac{1}{2} R_1]{} \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 3 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[\substack{R_2 \to -3R_1 + R_2 \\ R_3 \to -2R_1 + R_3}]{} \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & -\frac{3}{2} & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_2 \to 2R_2]{} \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -1 & 0 \\ 0 & 1 & -1 & -3 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \xrightarrow[R_1 \to -\frac{1}{2} R_2 + R_1]{} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & -1 & -3 & 2 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[\substack{R_1 \to -R_3 + R_1 \\ R_2 \to R_3 + R_2}]{} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & -1 \\ 0 & 1 & 0 & -4 & 2 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

Lineær algebra

Likningsløsning vha invers matrise:

$$AX = b$$

$$A^{-1}AX = A^{-1}b$$

$$X = A^{-1}b$$

En ortogonal matrise er definert ved følgende egenskap: $Q^T Q = QQ^T = I$

For ortogonale matriser er den transponerte lik den inverse $Q^T = Q^{-1}$

Lineær algebra

$$X = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$T(t_x, t_y) = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$T(t_x, t_y)X = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x + t_x \\ y + t_y \\ 1 \end{bmatrix}$$

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$