Kap 25 Differentiallikninger - Løsningsforslag

01. Differentiallikninger:

a)

$$\frac{dy}{dx} = 4x$$

Video

$$dy = 4x dx$$
$$\int dy = \int 4x dx$$
$$\underline{\underline{y = 2x^2 + C}}$$

b)

$$\frac{dy}{dx} - 3x^2 = 0$$

Video

$$dy = 3x^2 dx$$
$$\int dy = \int 3x^2 dx$$
$$\underline{\underline{y = x^3 + C}}$$

c)

$$y' + 5y = 0$$
$$\frac{dy}{dx} = -5y$$
$$\frac{dy}{y} = -5dx$$

Video

$$\int \frac{dy}{y} = -5\int dx$$
$$\ln|y| = -5x + C$$
$$e^{\ln|y|} = e^{-5x+C} = e^{-5x} \cdot e^{C} = Ke^{-5x}$$
$$\underline{\underline{y = Ae^{-5x}}}$$

d)

$$y' - 5xy = 0$$

$$\frac{dy}{dx} = 5xy$$

$$\frac{dy}{y} = 5x\,dx$$

$$\int \frac{dy}{y} = 5\int x\,dx$$

$$\ln|y| = 5 \cdot \frac{1}{2}x^2 + c_1$$

$$\ln|y| = \frac{5}{2}x^2 + c_1$$

$$e^{\ln|y|} = e^{\frac{5}{2}x^2 + c_1}$$

$$|y| = e^{\frac{5}{2}x^2 + c_1} = e^{\frac{5}{2}x^2} e^{c_1} = c_2 e^{\frac{5}{2}x^2}$$

$$y = \pm c_2 e^{\frac{5}{2}x^2}$$

$$\underline{y = ce^{\frac{5}{2}x^2}}$$

Video

e)

$$y' - 5y = 2$$

$$\mu(x) = \int p(x)dx = \int (-5)dx = -5x$$

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$$y(x) = e^{-\mu(x)} \int e^{u(x)} q(x)dx$$

$$= e^{5x} \int e^{-5x} \cdot 2dx = 2e^{5x} \int e^{-5x} dx = 2e^{5x} \left[\frac{1}{-5} e^{-5x} + C \right]$$

$$= \underline{\underline{Ae^{5x} - \frac{2}{5}}}$$

f)

$$y''-4y'+2y=0$$

$$y=Ce^{rx}$$

$$y'=rCe^{rx}$$

$$y''=r^2Ce^{rx}$$

$$r^2Ce^{rx}-4rCe^{rx}+2Ce^{rx}=0$$

$$r^2-4r+2=0$$

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$$r=\frac{4\pm\sqrt{4^2-4\cdot1\cdot2}}{2}=\frac{4\pm2\sqrt{3}}{2}=2\pm\sqrt{3}$$

$$y_1=e^{(2-\sqrt{3})x}$$

$$y_2=e^{(2+\sqrt{3})x}$$

$$y=Ay_1+By_2=\underline{Ae^{(2-\sqrt{3})x}+Be^{(2+\sqrt{3})x}}$$

02.

$$x(t)=A\cos(\omega t+\varphi) \qquad \omega=2\pi f=\sqrt{\frac{k}{m}} \qquad T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{m}{k}} \qquad \varphi=\tan^{-1}\left(-\frac{v_0}{\omega x_0}\right)$$

$$A=x_0=0.50 \qquad \omega=\sqrt{\frac{2.0}{0.2}}=\underline{\sqrt{10}} \qquad T=\frac{2\pi}{\sqrt{10}} \qquad f=\frac{1}{T}=\frac{\sqrt{10}}{2\pi} \qquad \varphi=\tan^{-1}\left(-\frac{0}{4\pi\cdot0.20}\right)=0$$

Video