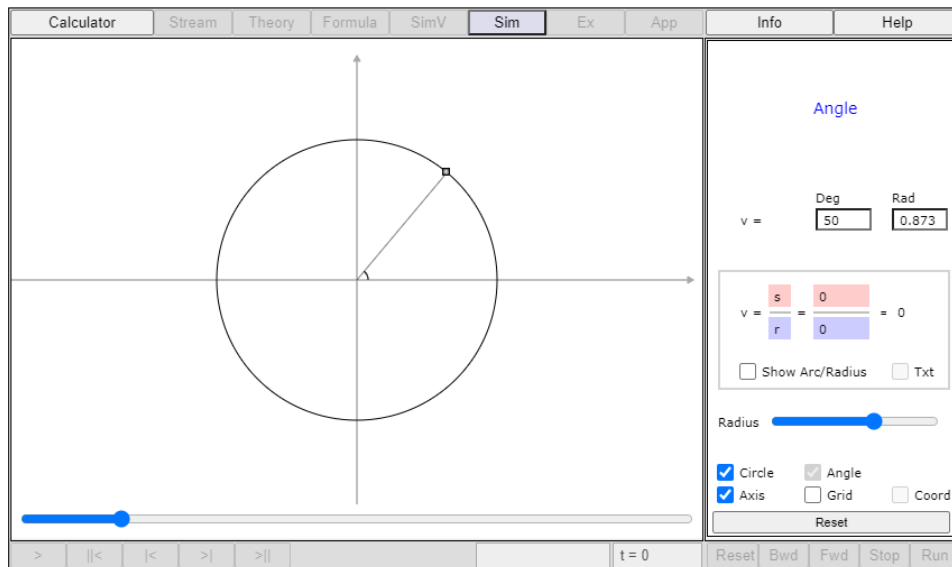


Trigonometri - Løsningsforslag

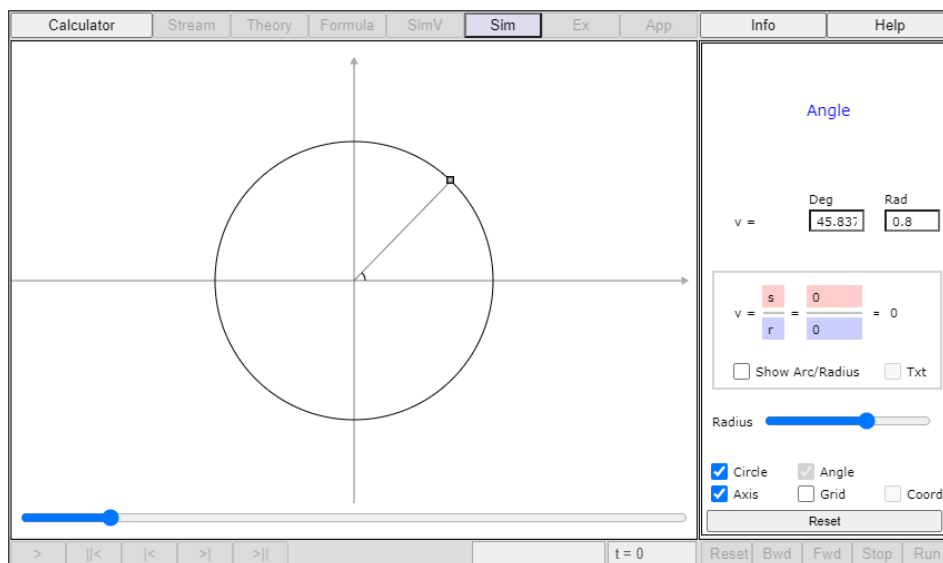
00. a) Omgjør 50 grader til radianer.

$$\alpha = \frac{\pi}{180^\circ} 50^\circ = \frac{5}{18} \pi = 0.87266... \approx \underline{\underline{0.8727}}$$



b) Omgjør 0.8 radianer til grader.

$$\alpha = \frac{180^\circ}{\pi} 0.8 = 45.83332...^\circ \approx \underline{\underline{45.8333^\circ}}$$

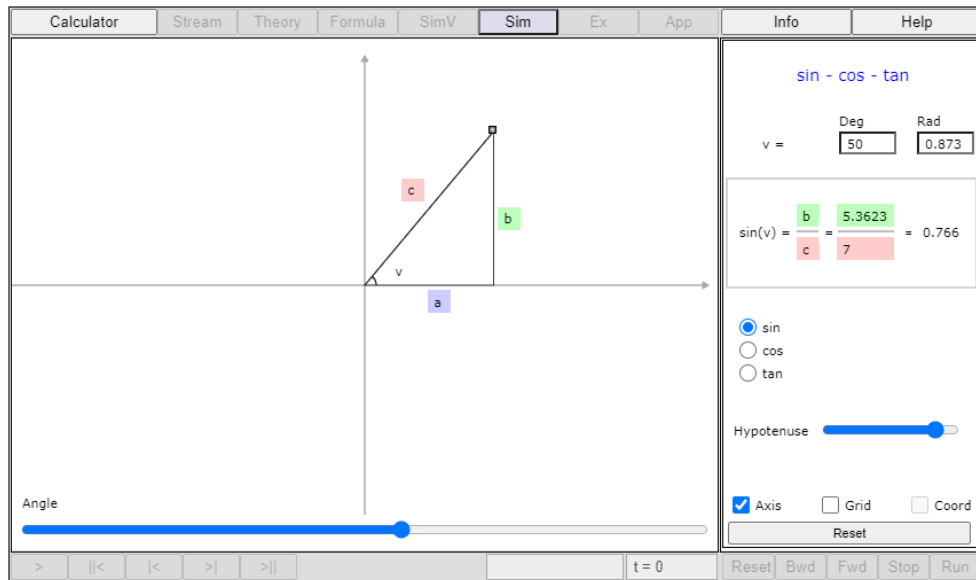


c) Regn ut sinus, cosinus og tangens til 50 grader.

$$\sin(50^\circ) = \underline{\underline{0.7660\dots}}$$

$$\cos(50^\circ) = \underline{\underline{0.6427\dots}}$$

$$\tan(50^\circ) = \underline{\underline{1.1917\dots}}$$

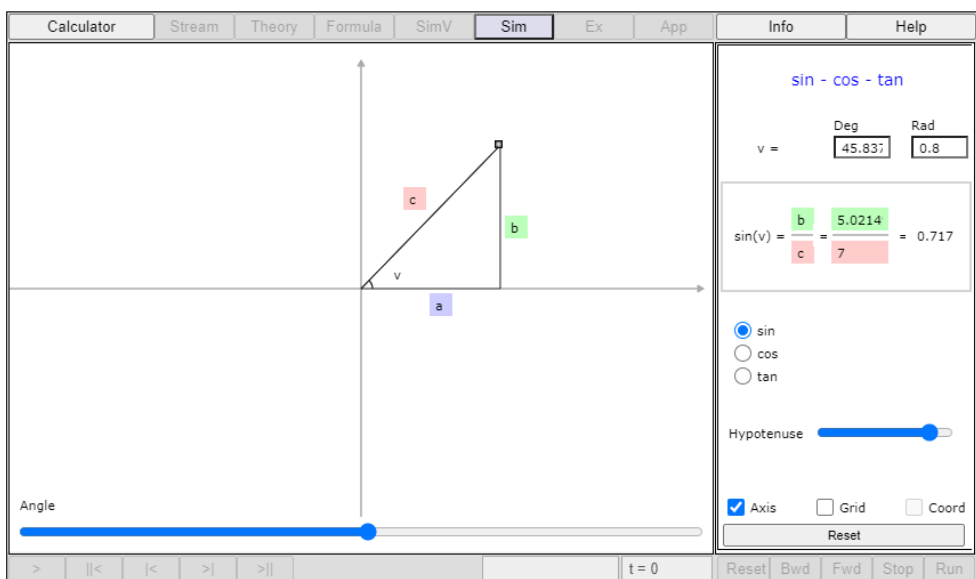


d) Regn ut sinus, cosinus og tangens til 0.8 radianer.

$$\sin(0.8) = \underline{\underline{0.7173\dots}}$$

$$\cos(0.8) = \underline{\underline{0.6967\dots}}$$

$$\tan(0.8) = \underline{\underline{1.0296\dots}}$$



e) Løs ligningen $5\sin(x) - 2 = 0$

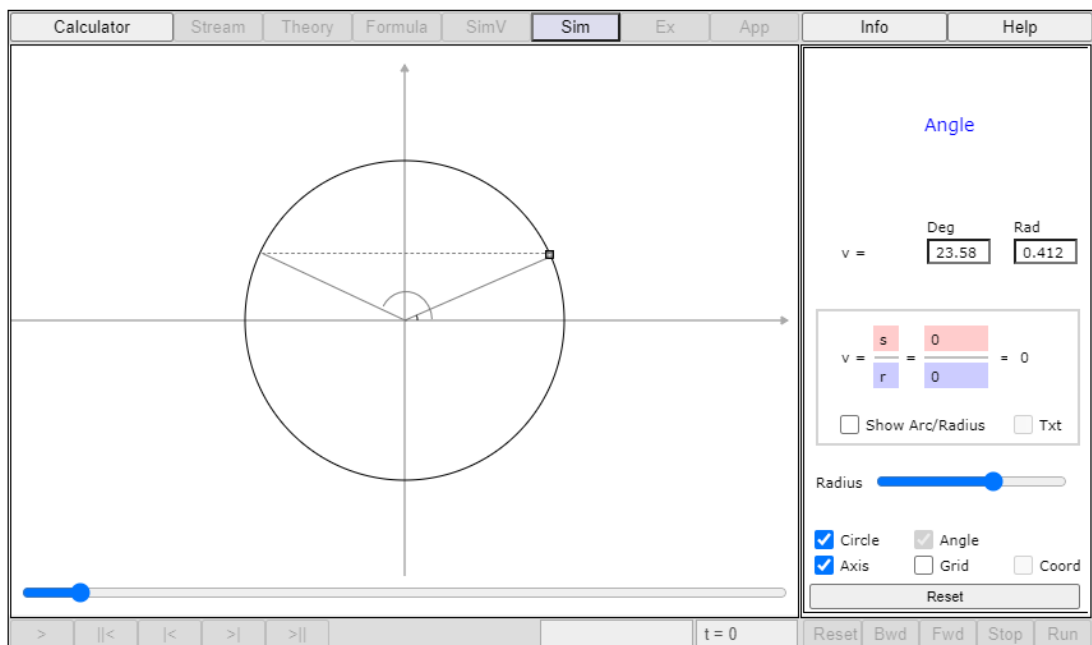
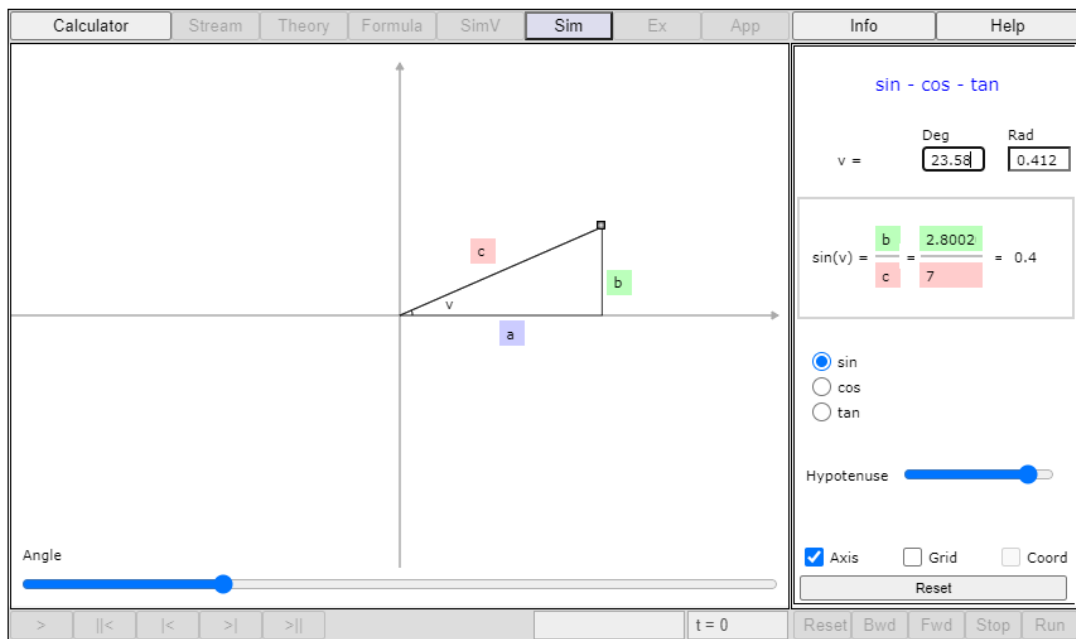
$x \in [0, 360^\circ]$

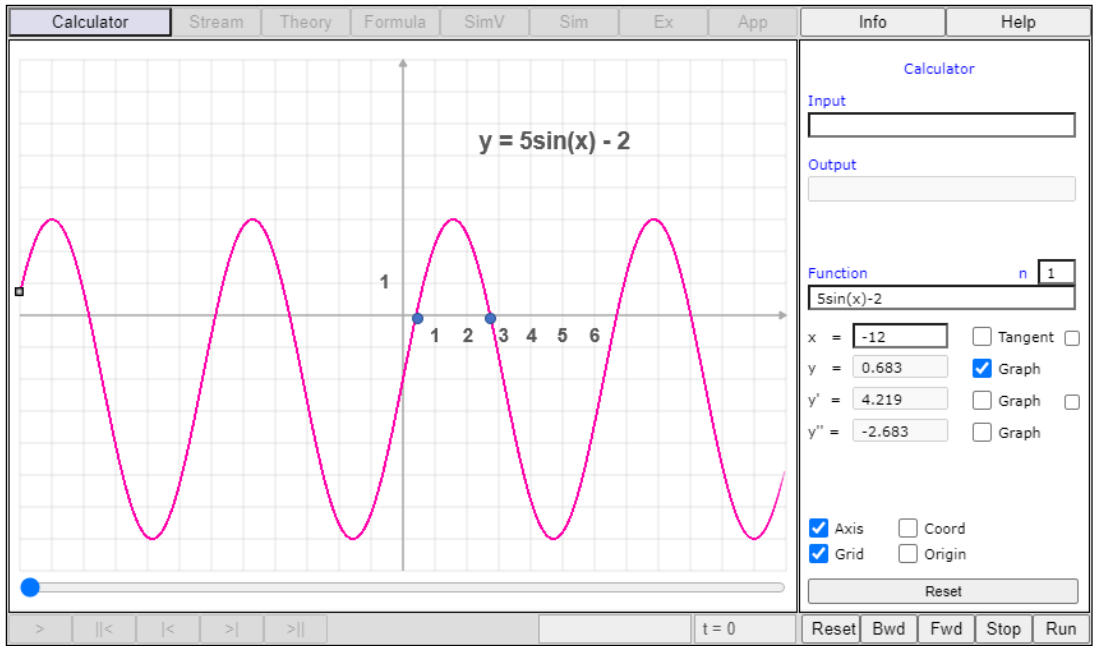
$$5\sin(x) - 2 = 0 \Rightarrow \sin(x) = \frac{2}{5}$$

⇓

$$x = \sin^{-1}\left(\frac{2}{5}\right) \Rightarrow x = \underline{0.4115} \quad \vee \quad x = \pi - 0.4115 = \underline{2.7301}$$

$$x = \underline{23.58^\circ} \quad \vee \quad x = 180^\circ - 23.58^\circ = \underline{156.42^\circ}$$





f) Løs ligningen $5\sin(2x) - 2 = 0$

$x \in [0, 360^\circ]$

$$5\sin(2x) - 2 = 0 \Rightarrow \sin(2x) = \frac{2}{5}$$

⇓

$$2x = \sin^{-1}\left(\frac{2}{5}\right) + k \cdot 2\pi \quad \vee \quad 2x = \pi - \sin^{-1}\left(\frac{2}{5}\right) + k \cdot 2\pi$$

$$x = \frac{1}{2}\sin^{-1}\left(\frac{2}{5}\right) + k \cdot \pi \quad \vee \quad x = \frac{\pi}{2} - \frac{1}{2}\sin^{-1}\left(\frac{2}{5}\right) + k \cdot \pi$$

⇓

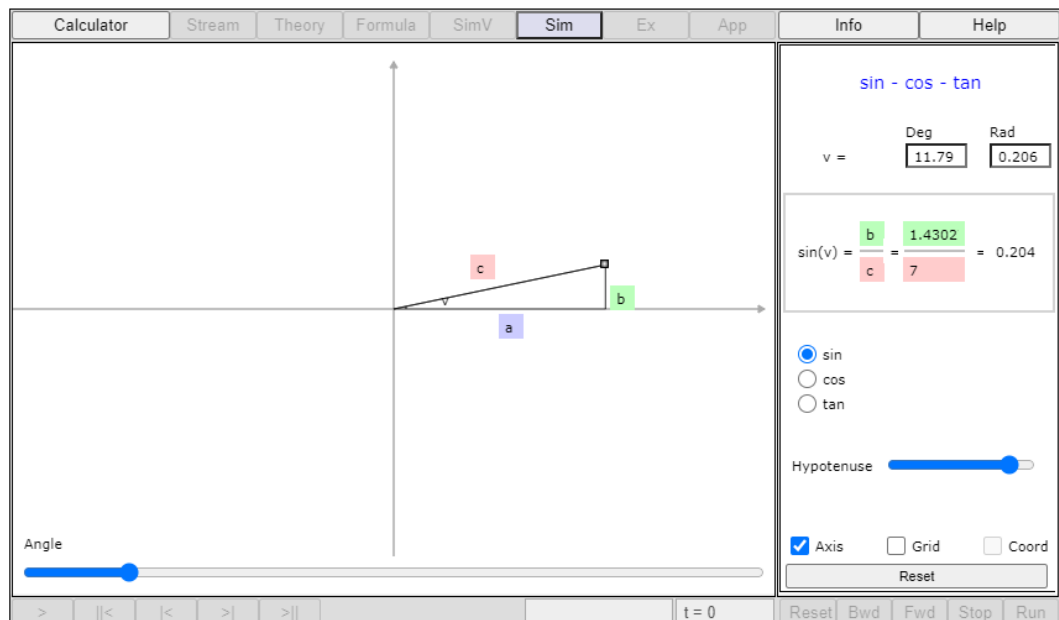
$$x = 0.2058 + k \cdot \pi \quad \vee \quad x = \frac{\pi}{2} - 0.2058 + k \cdot \pi$$

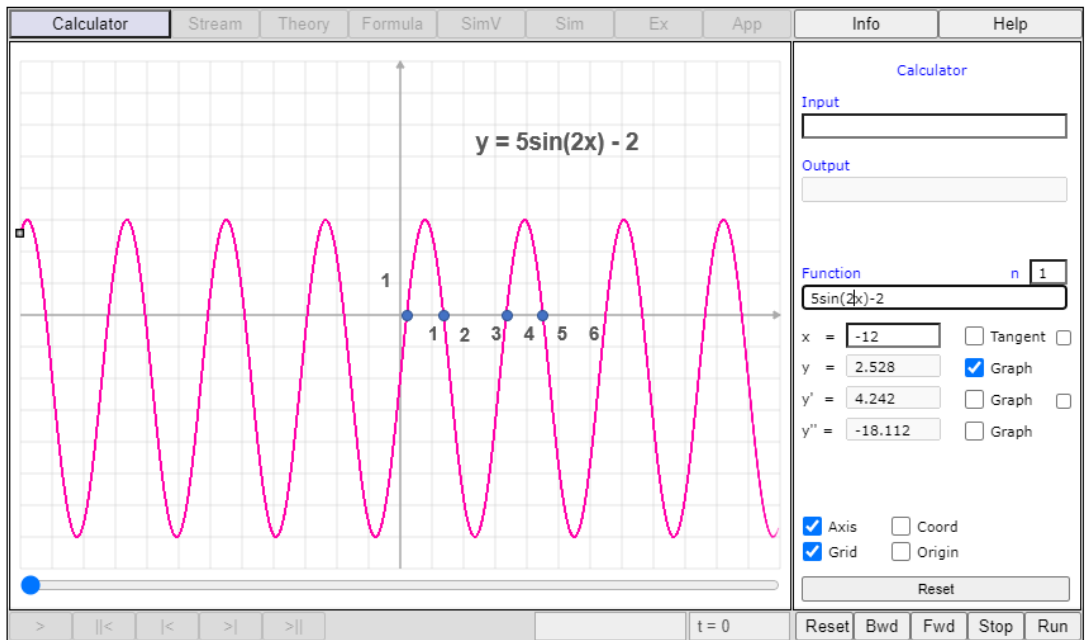
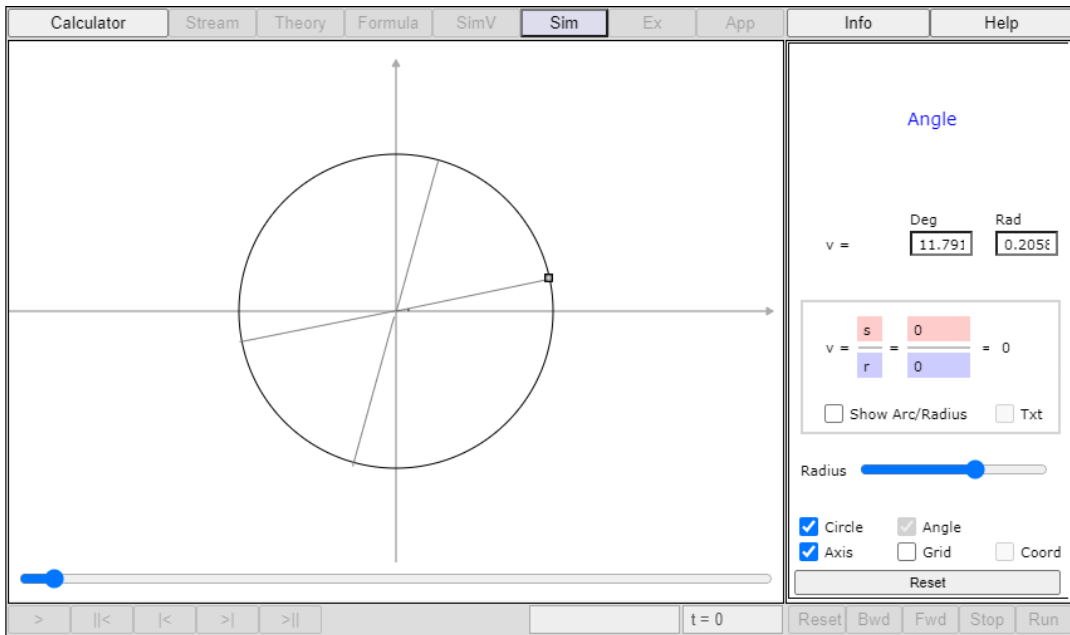
$$x = \underline{0.2058} \quad \vee \quad x = \frac{\pi}{2} - 0.2058 = \underline{1.3650} \quad k = 0$$

$$x = \underline{11.79^\circ} \quad \vee \quad x = \underline{78.21^\circ}$$

$$x = \underline{3.3474} \quad \vee \quad x = \frac{\pi}{2} - 3.3474 = \underline{4.5066} \quad k = 1$$

$$x = \underline{191.79^\circ} \quad \vee \quad x = \underline{258.21^\circ}$$





g) Løs ligningen $(5\sin(x) - 2)(2 - \cos(2x)) = 0$ $x \in [0, 360^\circ]$

$$(5\sin(x) - 2) \cdot (2 - \cos(2x)) = 0$$

⇓

$$5\sin(x) - 2 = 0 \quad \vee \quad 2 - \cos(2x) = 0$$

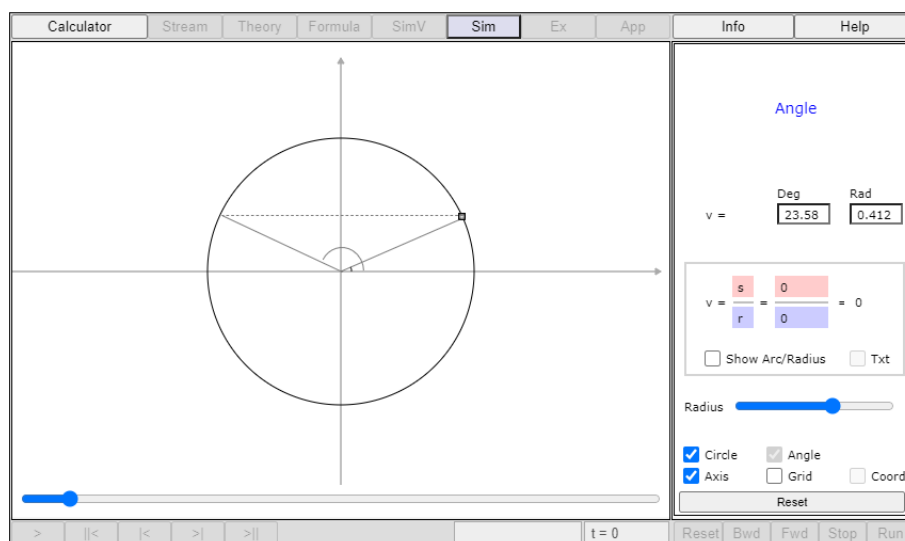
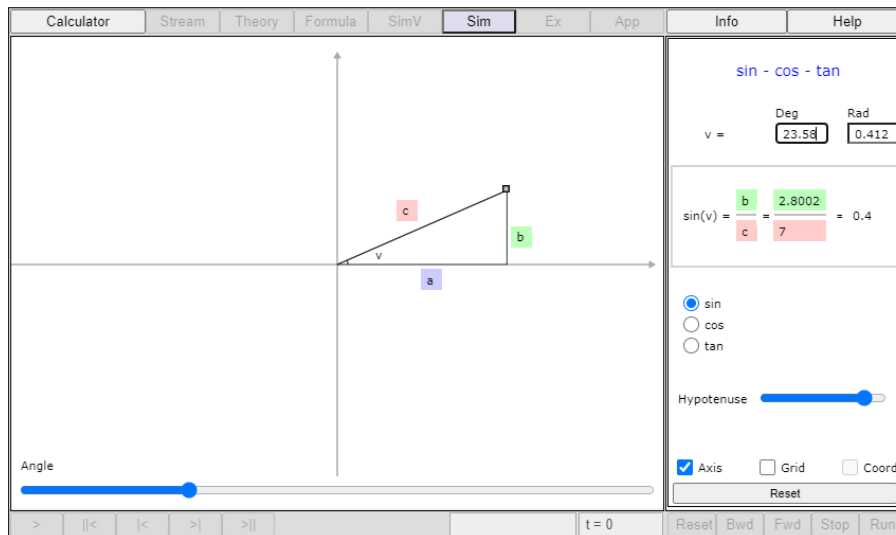
$$\sin(x) = \frac{2}{5} \quad \vee \quad \cos(2x) = 2$$

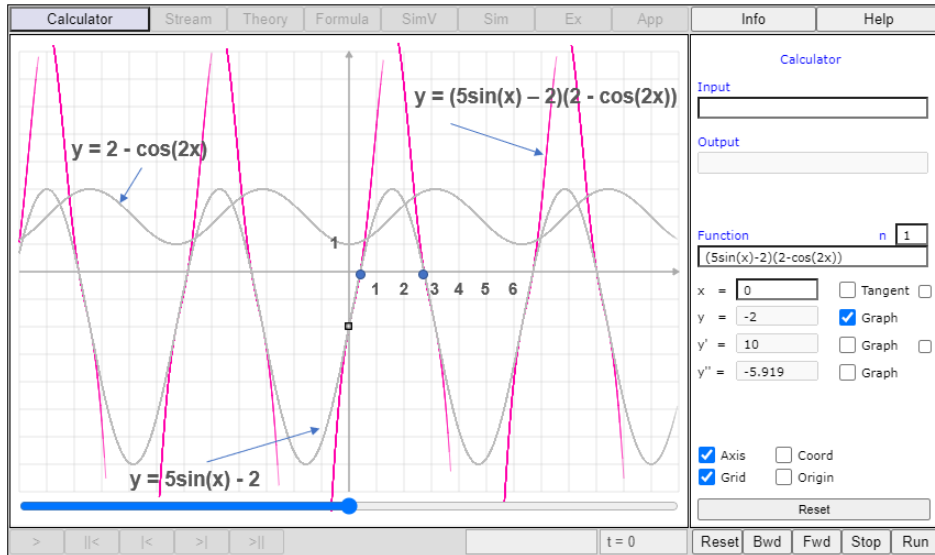
$$x = \sin^{-1}\left(\frac{2}{5}\right) \Rightarrow x = \underline{0.4115} \quad \vee \quad x = \pi - 0.4115 = \underline{2.7301}$$

$$x = \underline{23.58^\circ} \quad \vee \quad x = 180^\circ - 23.58^\circ = \underline{156.42^\circ}$$

$\cos(2x) = 2$ har ingen løsning.

cosinus til en vinkel kan ikke bli større enn 1.





h) Løs ligningen $2\sin(x) - 3\cos(x) = 0$ $x \in [0, 360^\circ]$

$$2\sin(x) - 3\cos(x) = 0 \quad : \cos(x) \quad \cos(x) \neq 0$$

↓

$$2 \frac{\sin(x)}{\cos(x)} - 3 \frac{\cos(x)}{\cos(x)} = \frac{0}{\cos(x)}$$

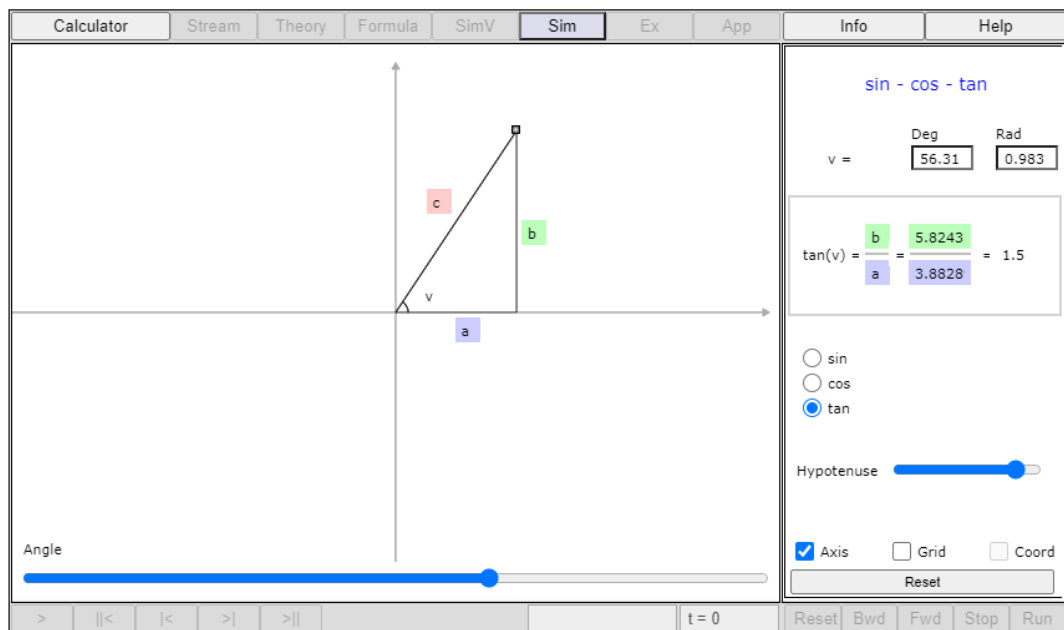
$$2 \tan(x) - 3 = 0$$

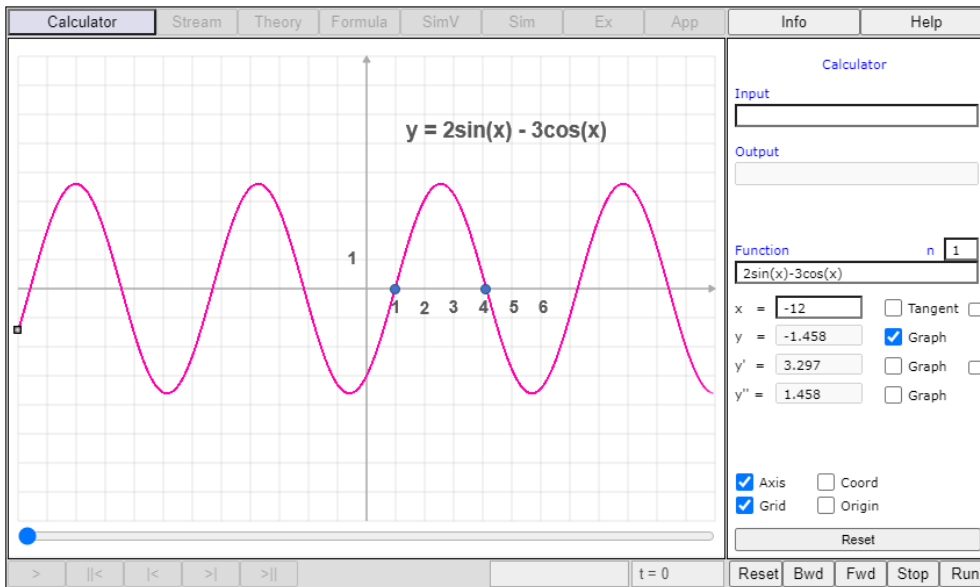
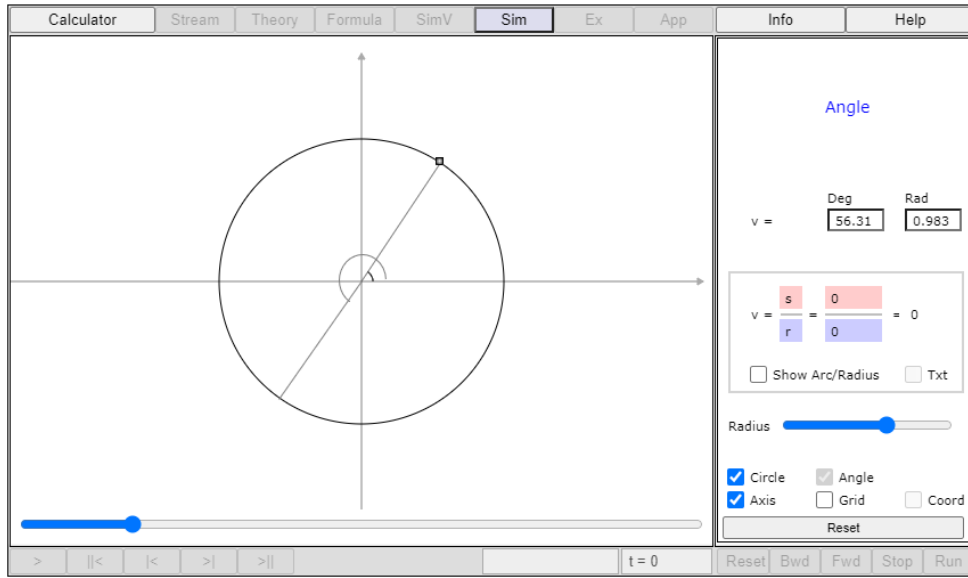
$$\tan(x) = \frac{3}{2}$$

$$x = \tan^{-1}\left(\frac{3}{2}\right)$$

$$x = \underline{0.9828} \quad \vee \quad x = \pi + 0.9828 = \underline{4.1244}$$

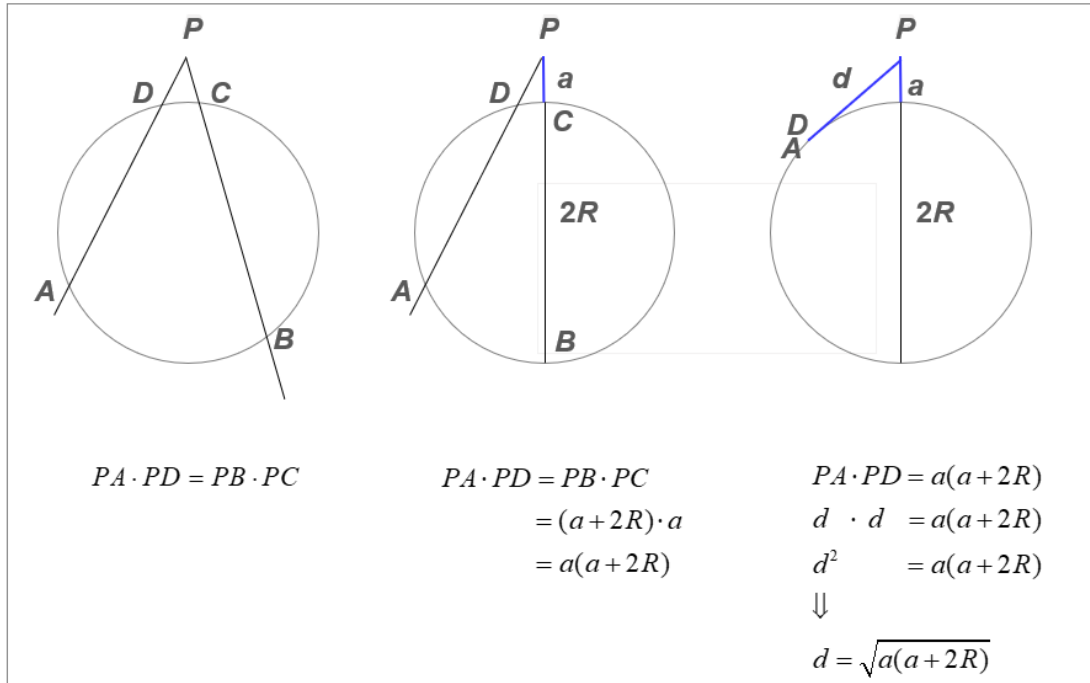
$$x = \underline{56.31^\circ} \quad \vee \quad x = 180^\circ + 56.31^\circ = \underline{236.31^\circ}$$





i) Beregn hvor langt du ser utover havet når du befinner seg 100 meter over havoverflaten.

Bruker et punkt potens mht en sirkel:



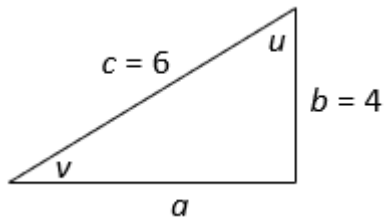
$$d = \sqrt{a(a + 2R)} = \sqrt{100 \text{ m} \cdot (100 \text{ m} + 2 \cdot 6400 \text{ km})}$$

Med $a = 100 \text{ m}$ og $R = 6400 \text{ km}$ får vi tilnærmet:

$$d = \sqrt{a(a + 2R)} \approx \sqrt{2aR}$$

$$= \sqrt{2 \cdot 100 \text{ m} \cdot 6400 \text{ km}} = \sqrt{2 \cdot 100 \frac{1}{1000} \text{ km} \cdot 6400 \text{ km}} \approx \underline{\underline{35.8 \text{ km}}}$$

01. Beregning av sider og vinkler i trekanten:



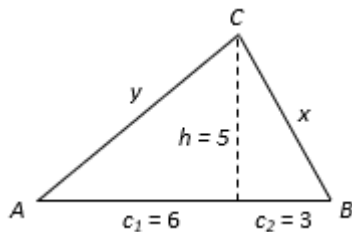
$$\sin v = \frac{b}{c} \Rightarrow v = \sin^{-1}\left(\frac{b}{c}\right) = \sin^{-1}\left(\frac{4}{6}\right) = \sin^{-1}\left(\frac{2}{3}\right) = 0.73 = \underline{\underline{41.8^\circ}}$$

$$u + v = 90^\circ \Rightarrow u = 90^\circ - v = 90^\circ - 41.8^\circ = \underline{\underline{48.2^\circ}}$$

$$\cos v = \frac{a}{c} \Rightarrow a = c \cdot \cos v = 6 \cdot \cos 0.73 \quad (= 6 \cdot \cos 41.8^\circ) = \underline{\underline{4.47}}$$

Video

02. Beregning av vinkler og sider i trekanten:



$$\tan A = \frac{h}{c_1} \Rightarrow A = \tan^{-1}\left(\frac{h}{c_1}\right) = \tan^{-1}\left(\frac{5}{6}\right) = 0.69 = \underline{\underline{39.8^\circ}}$$

$$\tan B = \frac{h}{c_2} \Rightarrow B = \tan^{-1}\left(\frac{h}{c_2}\right) = \tan^{-1}\left(\frac{5}{3}\right) = 1.03 = \underline{\underline{59.0^\circ}}$$

$$A + B + C = 180 \Rightarrow C = 180 - A - B = 180^\circ - 39.8^\circ - 59.0^\circ = \underline{\underline{81.2^\circ}}$$

Video

03. Trigonometrisk likning:

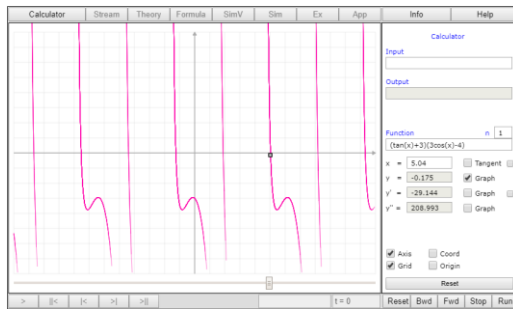
$$(\tan x + 3)(3 \cos x - 4) = 0 \quad x \in [180^\circ, 360^\circ]$$

$$\tan x + 3 = 0 \quad \vee \quad 3 \cos x - 4 = 0$$

$$\tan x + 3 = 0 \quad \vee \quad 3 \cos x = 4$$

$$\tan x = -3 \quad \vee \quad \cos x = \frac{4}{3}$$

$$\underline{\underline{x = 288,4^\circ}} \quad \vee \quad \text{Ingen l\u00f8sning fra siste faktor siden vi alltid har } |\cos x| \leq 1$$

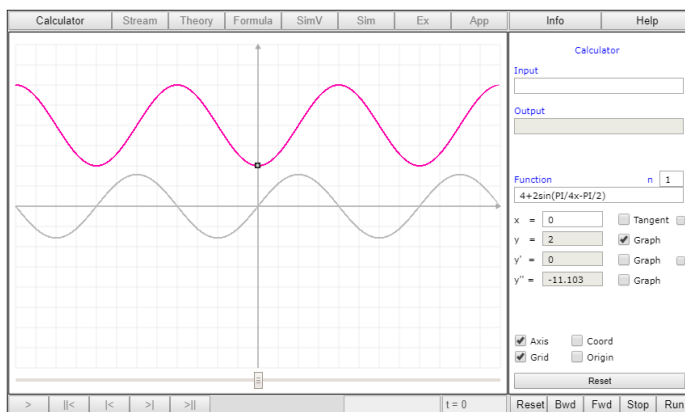


Video

04. Derivasjon:

$$f(x) = 4 + 2 \sin\left(\frac{\pi}{4}x - \frac{\pi}{2}\right)$$

$$f'(x) = 2 \cos\left(\frac{\pi}{4}x - \frac{\pi}{2}\right) \cdot \frac{\pi}{4} = \underline{\underline{\frac{\pi}{2} \cos\left(\frac{\pi}{4}x - \frac{\pi}{2}\right)}}$$



Video

05. Vi har gitt følgende funksjon: $f(x) = \sin x$ $x \in [0, \pi]$

a) Amplituden er lik 1

Perioden er lik 2π

Likevektslinjen er x -aksen ($y = 0$)

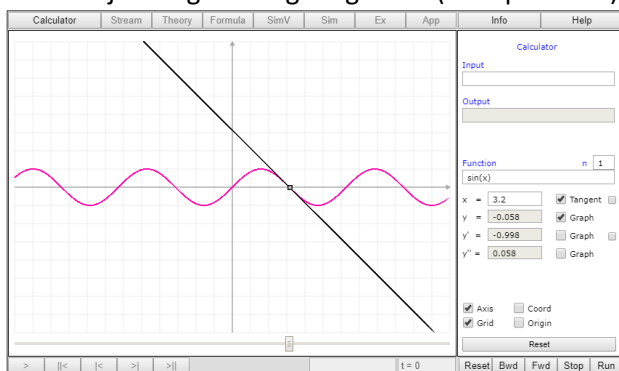
b) Likningen for tangenten i punktet $(0, \pi)$:

$$a = f'(\pi) = \cos(\pi) = -1$$

$$y - 0 = -1(x - \pi)$$

$$\underline{\underline{y = -x + \pi}}$$

Illustrasjon av grafen og tangenten (klikk på bildet):



c) Arealet som er avgrenset av grafen til f og x -aksen:

$$\text{Arealet} = A = \int_0^{\pi} \sin x \, dx = -[\cos x]_0^{\pi} = -(-1 - 1) = \underline{\underline{2}}$$

d) Volum av omdreiningslegemet:

$$V = \pi \int_0^{\pi} f^2(x) \, dx = \pi \int_0^{\pi} \sin^2(x) \cdot dx = \pi \int_0^{\pi} \frac{1 - \cos(2x)}{2} \cdot dx = \underline{\underline{\frac{1}{2} \pi^2}}$$

Video

06. a) Amplitude, likevektslinje, periode og faseforskyvning:

$$f(x) = 4 - \sin\left(2x - \frac{\pi}{2}\right) \quad x \in [0, 2\pi]$$

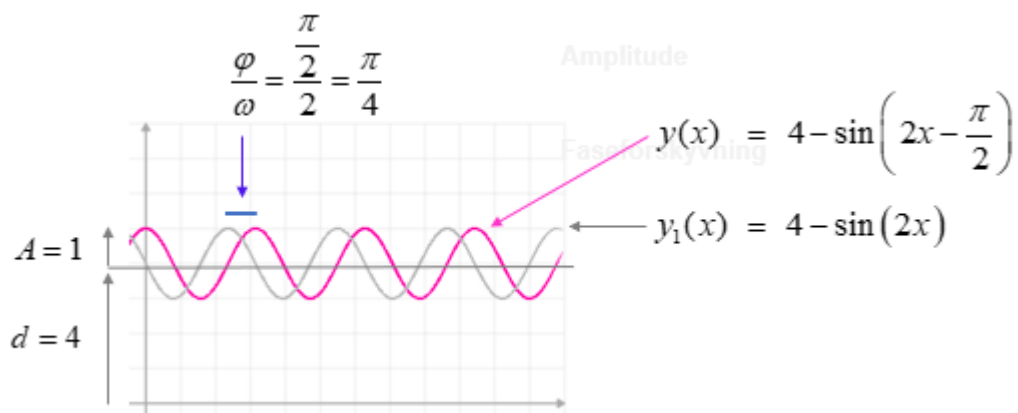
$$f(x) = 4 - 1 \cdot \sin\left[2\left(x - \frac{\pi}{4}\right)\right]$$

Amplituden er 1

Likevektslinjen er $y = 4$

$$\text{Perioden er } T = \frac{2\pi}{2} = \pi$$

Faseforskyvning $\varphi = \frac{\pi}{4}$ mot høyre



b) Maks-, min-punkter:

$$f_{\max} = 5 \text{ når } \sin(2x - \pi/2) = -1 \Rightarrow 2x - \pi/2 = 3\pi/2 \Rightarrow \underline{x = \pi}$$

Vi får da 3 maksimumspunkter, nemlig:

$$\underline{(0, 5), (\pi, 5) \text{ og } (2\pi, 5)}$$

$$f_{\min} = 3 \text{ når } \sin(2x - \pi/2) = 1.$$

Vi vet at min-punktene ligger midt mellom maks-punktene:

Vi får da 3 minimumspunkter, nemlig:

$$\underline{(\pi/2, 3) \text{ og } (3\pi/2, 3)}$$

c) Vendepunkter:

Vendepunktene ligger midt mellom et maks-punkt og et min-punkt.

I tillegg ligger vendepunktene på likevektslinjen.

Vi får da 4 vendepunkter, nemlig:

$$\underline{\underline{\left(\frac{\pi}{4}, 4\right), \left(\frac{3\pi}{4}, 4\right), \left(\frac{5\pi}{4}, 4\right) \text{ og } \left(\frac{7\pi}{4}, 4\right)}}$$

d) Tangent:

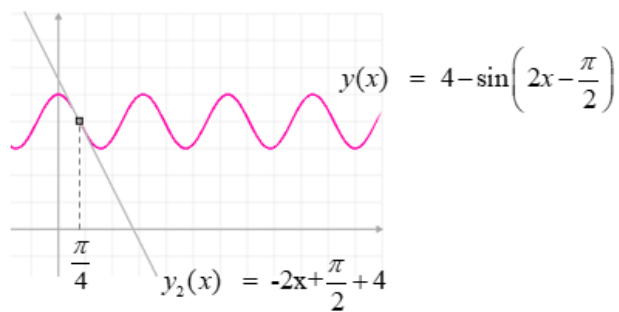
$$\underline{\underline{f\left(\frac{\pi}{4}\right) = 4 - \sin\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = 4}}$$

$$\underline{\underline{f'(x) = -2\cos\left(2x - \frac{\pi}{2}\right) \Rightarrow f'\left(\frac{\pi}{4}\right) = -2\cos\left(\frac{\pi}{2} - \frac{\pi}{2}\right) = -2}}$$

Ett-punkts formelen gir da:

$$y - 4 = -2\left(x - \frac{\pi}{4}\right) = -2x + \frac{\pi}{2}$$

$$\underline{\underline{y = -2x + \frac{\pi}{2} + 4 \approx -2x + 5,57}}$$



Video

07. Likninger:

a)

$$5 \cos v = 1 + 3 \cos v \quad v \in [0, 360^\circ >$$

$$5 \cos v = 1 + 3 \cos v$$

$$2 \cos v = 1$$

$$\cos v = \frac{1}{2}$$

$$\underline{\underline{v = 60^\circ \quad \text{eller} \quad v = 300^\circ}}$$

b)

$$20 \sin^2 x - 9 \cos x = 19 \quad x \in [0, 360^\circ >$$

$$20 \sin^2 x - 9 \cos x = 19$$

$$20(1 - \cos^2 x) - 9 \cos x - 19 = 0$$

$$-20 \cos^2 x - 9 \cos x + 1 = 0$$

ABC formel på kalkulator gir:

$$\cos x_1 = -0,54 \quad \text{eller} \quad \cos x_2 = 0,09$$

$$\underline{\underline{x_1 = 122,6^\circ \quad \text{eller} \quad x_1 = 360^\circ - 122,6^\circ = 237,2^\circ}}$$

$$\underline{\underline{x_2 = 84,8^\circ \quad \text{eller} \quad x_2 = 360^\circ - 84,8^\circ = 275,2^\circ}}$$

Video