# Mathematics – Algebra - Exercises - Solutions

Click on the exercise number to start a video.

01 Evaluate:

**Evaluate:** $a^2 - 3ab - 2b^2$

**When a = -3, b = -2**

a) 19
b) -1
c) -17
d) -35
e) None of the above

Solution:

$$a^2 - 3ab - 2b^2 = (-3)^2 - 3\cdot(-3)\cdot(-2) - 2\cdot(-2)^2 = 9 - 18 - 8 = \underline{-17} \qquad \underline{\underline{c}}$$

02 Evaluate:

**Evaluate:** $a^2 - 3ab - 2b^2$

**When a = 2b**

a) 0
b) $-4b^2$
c) $-6b^2$
d) $-10b^2$
e) None of the above

Solution:

$$a^2 - 3ab - 2b^2 = (2b)^2 - 3\cdot(2b)\cdot b - 2b^2 = 4b^2 - 6b^2 - 2b^2 = \underline{-4b^2} \qquad \underline{\underline{b}}$$

03 Evaluate:

**Evaluate:** $a^2 - 3ab - 2b^2$

**When b = -3a**

a) $-8a^2$
b) $16a^2$
c) $-26a^2$
d) $-2a^2$
e) None of the above

Solution:

$$a^2 - 3ab - 2b^2 = a^2 - 3a\cdot(-3a) - 2(-3a)^2 = a^2 + 9a^2 - 18a^2 = \underline{-8a^2} \qquad \underline{\underline{a}}$$

04 Evaluate:

**If x = -1 and y = -3, evaluate: 3x - 2 [4 - x {6x - 5y + 3 (3y - 2x - 6) + 17}]**

a) 15
b) -21
c) -12
d) -15
e) None of the above

Solution:

$3x-2[4-x\{6x-5y+3(3y-2x-6)+17\}]=$

$3x-2[4-x\{6x-5y+9y-6x-18+17\}]=$

$3x-2[4-x\{4y-1\}]=$

$3x-2[4-4xy+x]=$

$3x-8+8xy-2x=$

$x+8xy-8=$

$(-1)+8\cdot(-1)\cdot(-3)-8=$

$-1+24-8=$

$\underline{\underline{15}}$ $\underline{\underline{a}}$

or

$3x-2[4-x\{6x-5y+3(3y-2x-6)+17\}]=$

$3\cdot(-1)-2[4-(-1)\cdot\{6\cdot(-1)-5\cdot(-3)+3\cdot(3\cdot(-3)-2\cdot(-1)-6)+17\}]=$

$-3-2[4+1\cdot\{-6+15+3\cdot(-9+2-6)+17\}]=$

$-3-2[4+\{-6+15-39+17\}]=$

$-3-2[4-13]=$

$-3-2\cdot(-9)=$

$-3+18=$

$\underline{\underline{15}}$ $\underline{\underline{a}}$

05 Polynomial - Addition:

**Find the sum of the polynomials:** $(x^3 + 3x^2y - 3y^3)$ , $(x^2y - 2xy^2 + y^3)$ , $(3x^3 - 4x^2y + 3xy^2 - 2y^3)$

a) $4x^3 + x^2y + xy^2 - 4y^3$
b) $4x^3 - x^2y + xy^2 - 4y^3$
c) $4x^3 - xy^2 - 4y^3$
d) $4x^3 + xy^2 - 4y^3$
e) None of the above

Solution:

$$(x^3 + 3x^2y - 3y^3) + (x^2y - 2xy^2 + y^3) + (3x^3 - 4x^2y + 3xy^2 - 2y^3) =$$
$$x^3 + 3x^2y - 3y^3 + x^2y - 2xy^2 + y^3 + 3x^3 - 4x^2y + 3xy^2 - 2y^3 =$$
$$\underline{4x^3 + xy^2 - 4y^3} \qquad \underline{\underline{d}}$$

06 Polynomial – Multiplication

**Perform the multiplication:**

$(x^2 + y^2 - xy)\ (x^2 + xy - y^2)$

a) $x^4 - x^2y^2 + 2xy^3 - y^4$
b) $x^4 - x^3y - x^2y^2 + 2xy^3 - y^4$
c) $x^4 + 2xy^3 - y^4$
d) $x^4 - x^2y^2 - y^4$
e) None of the above

Solution:

$$(x^2 + y^2 - xy)(x^2 + xy - y^2) =$$
$$x^4 + x^2y^2 - x^3y + x^3y + xy^3 - x^2y^2 - x^2y^2 - y^4 + xy^3 =$$
$$\underline{x^4 - x^2y^2 + 2xy^3 - y^4} \qquad \underline{\underline{a}}$$

07 Polynomial – Multiplication

**Perform the multiplication:**

**$(3x - 4y)^2$**

a) $9x^2 + 24xy + 16y^2$
b) $9x^2 + 16y^2$
c) $9x^2 - 16y^2$
d) $9x^2 - 24xy + 16y^2$
e) None of the above

Solution:

$$(3x-4y)^2 =$$
$$(3x)^2 - 2\cdot 3x\cdot 4y + (4y)^2 =$$
$$\underline{9x^2 - 24xy + 16y^2} \qquad \underline{\underline{d}}$$

08 Division:

**Perform the division: $(16y^4 - 1) / (2y + 1)$**

a) $8y^3 - 4y^2 - 2y + 1$
b) $8y^3 + 4y^2 + 2y - 1$
c) $8y^3 - 4y^2 + 2y - 1$
d) $8y^3 + 4y^2 + 2y + 1$
e) None of the above

Solution:

$$\frac{16y^4 - 1}{2y+1} =$$
$$\frac{(4y^2-1)(4y^2+1)}{2y+1} =$$
$$\frac{(2y-1)(2y+1)(4y^2+1)}{2y+1} =$$
$$(2y-1)(4y^2+1) =$$
$$\underline{8y^3 - 4y^2 + 2y - 1} \qquad \underline{\underline{c}}$$

09 Factor:

**Factor:**

**$(a + 2b)^2 - (b - 2a)^2$**

a) $(3a + b)(a - 3b)$
b) $(3a + b)(3b - a)$
c) $(3a - b)(3b - a)$
d) $(3a + b)(3b + a)$
e) None of the above

Solution:

$$(a+2b)^2-(b-2a)^2=$$
$$[(a+2b)-(b-2a)]\cdot[(a+2b)+(b-2a)]=$$
$$[a+2b-b+2a]\cdot[a+2b+b-2a]=$$
$$[3a+b]\cdot[3b-a]=$$
$$\underline{(3a+b)(3b-a)} \qquad \underline{\underline{b}}$$

10 Factor:

**Factor:**

**$16a^4 - 24a^2bc^3 + 9b^2c^6$**

a) $(4a^2 - 3b^2)(4a^2 - 3c^6)$
b) $(4a^2 + 3bc^3)^2$
c) $(4a^2 + 3b^2)(4a^2 + 3c^6)$
d) $(4a^2 - 3bc^3)^2$
e) None of the above

Solution:

$$16a^4-24a^2bc^3+9b^2c^6=(4a^2)^2-2\cdot 4a^2\cdot 3bc^3+(3bc^3)^2=\underline{(4a^2-3bc^3)^2} \qquad \underline{\underline{d}}$$

11 Factor:

**Factor:**

$x^6 - 64y^6$

a) $(x + 2y)(x - 2y)(x^2 + 2xy - 4y^2)(x^2 + 2xy + 4y^2)$
b) $(x - 2y)^2(x - 2xy + 4y^2)^2$
c) $(x + 2y)^2(x - 2xy + 4y^2)^2$
d) $(x + 2y)(x - 2y)(x^2 + 2xy + 4y^2)(x^2 - 2xy + 4y^2)$
e) None of the above

Solution:

Here we see that b) or c) cannot be a solution (don't give $x^6$).
Multiplication gives:

$$x^6 - 64y^6 = \underline{(x-2y)(x+2y)(x^2-2xy+4y^2)(x^2+2xy+4y^2)} \qquad \underline{\underline{d}}$$

or

We use: $(a^3 \pm b^3) = (a \pm b)(a^2 \mp ab + b^2)$

$$\begin{aligned} x^6 - 64y^6 &= (x^3)^2 - (8y^3)^2 \\ &= \left[x^3 - 8y^3\right]\left[x^3 + 8y^3\right] \\ &= \left[x^3 - (2y)^3\right]\left[x^3 + (2y)^3\right] \\ &= \left[(x-2y)(x^2 + x \cdot 2y + (2y)^2)\right]\left[(x+2y)(x^2 - x \cdot 2y + (2y)^2)\right] \\ &= \underline{(x+2y)(x-2y)(x^2+2xy+4y^2)(x^2-2xy+4y^2)} \qquad \underline{\underline{d}} \end{aligned}$$

12 Factor:

**Factor:**

**$2x^2$ - px - 2pf + 4fx**

a) (x + 2f)(2x - p)
b) (x - 2f)(2x + p)
c) (x + p)(2x - f)
d) (x - p)(2x + f)
e) None of the above

Solution:

Here we see quickly by multiplication that:

$$2x^2 - px - 2pf + 4fx = \underline{(x+2f)(2x-p)} \qquad \underline{\underline{a}}$$

or

We set *x* equal to a part of the constant term 2*pf*, for example x = -2*f*.
For this value of *x*, the expression is equal to zero.
Therefor *x+2f* is a factor of the expression.
Polynomial division gives the quotient 2*x-p*.
Therefor we have:

$$2x^2 - px - 2pf + 4fx = \underline{(x+2f)(2x-p)} \qquad \underline{\underline{a}}$$

13 Fraction:

**For what values of x is the following fraction not defined?** $\frac{3x^2}{x^2 + 2x - 3}$

a) 0
b) -1 and 3
c) 1 and -3
d) 1 and 3
e) None of the above

Solution:

$$x^2 + 2x - 3 = 0$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} = \frac{-2 \pm \sqrt{16}}{2 \cdot 1} = \frac{-2 \pm 4}{2 \cdot 1} = -1 \pm 2$$

$\underline{x = -3} \quad \vee \quad \underline{x = 1}$ c

14 Reduce a fraction:

**Reduce to lowest terms:** $\frac{a^3 - a^2 b + ab^2 - b^3}{b^4 - a^4}$

a) $\frac{1}{a + b}$
b) $- \frac{1}{a + b}$
c) $\frac{a - b}{a + b}$
d) $\frac{b - a}{a + b}$
e) None of the above

Solution:

$$\frac{a^3 - a^2 b + ab^2 - b^3}{b^4 - a^4} = \frac{a^3 - b^3 - ab(a - b)}{(b^2 - a^2)(b^2 + a^2)}$$

$$= \frac{(a - b)(a^2 + ab + b^2) - ab(a - b)}{(b^2 - a^2)(b^2 + a^2)}$$

$$= \frac{(a - b)(a^2 + ab + b^2 - ab)}{(b - a)(b + a)(b^2 + a^2)}$$

$$= -\frac{(a^2 + b^2)}{(b + a)(b^2 + a^2)}$$

$$= -\underline{\frac{1}{(a + b)}}$$ b

15 Change a fraction:

**Change $\frac{1}{2x - 8}$ into an equivalent fraction the denominator of which is $6(x^2 - 16)$:**

a) $\frac{3(x - 4)}{6(x^2-16)}$

b) $\frac{3(x + 4)}{6(x^2-16)}$

c) $\frac{3(x + 2)}{6(x^2-16)}$

d) $\frac{3(x - 2)}{6(x^2-16)}$

e) None of the above

Solution:

$$\frac{1}{2x-8} = \frac{1}{2(x-4)} = \frac{1 \cdot 3(x+4)}{2(x-4) \cdot 3(x+4)} = \underline{\frac{3(x+4)}{6(x^2-16)}}$$

b

16 Combine fractions:

**Combine into a single fraction:** $\frac{1}{(a - b)(a - c)} + \frac{1}{(b - a)(c - b)} - \frac{1}{(a - c)(b - c)}$

a) $\frac{2}{(a - b)(a - c)}$

b) $\frac{2}{(a - c)(b - c)}$

c) $\frac{2c}{(a - b)(a - c)(b - c)}$

d) $\frac{2b}{(a - b)(a - c)(b - c)}$

e) None of the above

Solution:

$$\frac{1}{(a-b)(a-c)}+\frac{1}{(b-a)(c-b)}-\frac{1}{(a-c)(b-c)}=$$

$$\frac{1}{(a-b)(a-c)}+\frac{1}{(a-b)(b-c)}-\frac{1}{(a-c)(b-c)}=$$

$$\frac{1\cdot(b-c)}{(a-b)(a-c)(b-c)}+\frac{1\cdot(a-c)}{(a-b)(b-c)(a-c)}-\frac{1\cdot(a-b)}{(a-c)(b-c)(a-b)}=$$

$$\frac{b-c+a-c-a+b}{(a-b)(a-c)(b-c)}=$$

$$=\frac{2(b-c)}{(a-b)(a-c)(b-c)}=\underline{\frac{2}{(a-b)(a-c)}}$$

a

17 Simplify:

**Simplify:**

$$\sqrt[5]{\sqrt{\frac{\sqrt[6]{f^{-6}}}{\sqrt[3]{f^{-1}}}}\;\sqrt[4]{f^{3}}}$$

a) f $^{1/5}$
b) f $^{1/12}$
c) f $^{17/60}$
d) f $^{3/240}$
e) None of the above

Solution:

Here we use: $\left(a^{m}\right)^{n}=a^{mn}$ $\quad a^{-1}=\frac{1}{a^{n}}$ $\quad \sqrt[q]{a^{p}}=a^{\frac{p}{q}}$ $\quad \sqrt{a}=\sqrt[2]{a}$

$$\sqrt[5]{\sqrt{\frac{\sqrt[6]{f^{-6}}}{\sqrt[3]{f^{-1}}}}\;\sqrt[4]{f^{3}}}=\left[\left(\frac{f^{\frac{-6}{6}}}{f^{\frac{-1}{3}}}\right)^{\frac{1}{2}} f^{\frac{3}{4}}\right]^{\frac{1}{5}}$$

$$=\left[\left(\frac{f^{-1}}{f^{-\frac{1}{3}}}\right)^{\frac{1}{2}} f^{\frac{3}{4}}\right]^{\frac{1}{5}}=\left[\left(f^{-1+\frac{1}{3}}\right)^{\frac{1}{2}} f^{\frac{3}{4}}\right]^{\frac{1}{5}}$$

$$=\left[\left(f^{-\frac{2}{3}}\right)^{\frac{1}{2}} f^{\frac{3}{4}}\right]^{\frac{1}{5}}=\left[f^{-\frac{2}{3}\cdot\frac{1}{2}} f^{\frac{3}{4}}\right]^{\frac{1}{5}}$$

$$=\left[f^{-\frac{1}{3}} f^{\frac{3}{4}}\right]^{\frac{1}{5}}=\left[f^{-\frac{1}{3}+\frac{3}{4}}\right]^{\frac{1}{5}}=\left[f^{\frac{5}{12}}\right]^{\frac{1}{5}}=f^{\frac{5}{12}\cdot\frac{1}{5}}=\underline{f^{\frac{1}{12}}}$$

b

18 Algebraic sum:

**Find the algebraic sum:** $\frac{1}{3}\sqrt{150} + \frac{5}{2}\sqrt{\frac{2}{75}} - 4\sqrt{\frac{3}{8}} - \frac{1}{4}\sqrt[4]{36}$

a) $\frac{5}{6}\sqrt{6}$

b) $\frac{15}{12}\sqrt{6}$

c) $\frac{1}{2}\sqrt{6}$

d) $\frac{7}{12}\sqrt{6}$

e) None of the above

Solution:

$$\frac{1}{3}\sqrt{150}+\frac{5}{2}\sqrt{\frac{2}{75}}-4\sqrt{\frac{3}{8}}-\frac{1}{4}\sqrt[4]{36}=$$

$$\frac{1}{3}\sqrt{6\cdot 25}+\frac{5}{2}\sqrt{\frac{2}{3\cdot 25}}-4\sqrt{\frac{3}{2\cdot 4}}-\frac{1}{4}\sqrt[4]{6^2}=$$

$$\frac{1}{3}\sqrt{6\cdot 5^2}+\frac{5}{2}\sqrt{\frac{2}{3\cdot 5^2}}-4\sqrt{\frac{3}{2\cdot 2^2}}-\frac{1}{4}6^{\frac{2}{4}}=$$

$$\frac{5}{3}\sqrt{6}+\frac{1}{2}\sqrt{\frac{2}{3}}-2\sqrt{\frac{3}{2}}-\frac{1}{4}6^{\frac{1}{2}}=$$

$$\frac{5}{3}\sqrt{6}+\frac{1}{2}\sqrt{\frac{2\cdot 3}{3\cdot 3}}-2\sqrt{\frac{3\cdot 2}{2\cdot 2}}-\frac{1}{4}\sqrt{6}=$$

$$\frac{5}{3}\sqrt{6}+\frac{1}{6}\sqrt{6}-\sqrt{6}-\frac{1}{4}\sqrt{6}=$$

$$\underline{\frac{7}{12}\sqrt{6}}$$ d

19 Simplify:

**Multiply and simplify:** $(\sqrt{5}-4\sqrt{2})(2\sqrt{2}+\sqrt{5})$

a) -11 + $2\sqrt{10}$
b) 11 - $2\sqrt{10}$
c) -11 - $2\sqrt{10}$
d) 11 + $2\sqrt{10}$
e) None of the above

Solution:

$$\left(\sqrt{5}-4\sqrt{2}\right)\left(2\sqrt{2}+\sqrt{5}\right)=\sqrt{5}\cdot 2\sqrt{2}-4\sqrt{2}\cdot 2\sqrt{2}+\sqrt{5}\cdot\sqrt{5}-4\sqrt{2}\cdot\sqrt{5}$$
$$=2\sqrt{10}-16+5-4\sqrt{10}$$
$$=\underline{-11-2\sqrt{10}} \qquad \underline{\underline{c}}$$

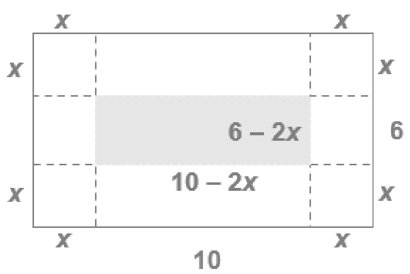
20 Volume of a box:

**An open box is to be made from a sheet of cardboard 6 inches by 10 inches by cutting equal squares of side length "x" inches from each of the four corners and bending up the sides.**

**Express the volume of the box, V, as a function of x**

a) V = 4x (3 - x)(5 - x)
b) V = 2x (3 - x)(5 - x)
c) V = x (6 - x)(10 - x)
d) V= 2x (6 - x)(10 - x)
e) None of the above

Solution:




The ground area will be a rectangle with sides 10-2*x* and 6-2*x*.
The height will be *x*.
Therefore the volume will be:

$$V = (10-2x)(6-2x)x = \underline{4x(5-x)(3-x)} \qquad \underline{\underline{a}}$$

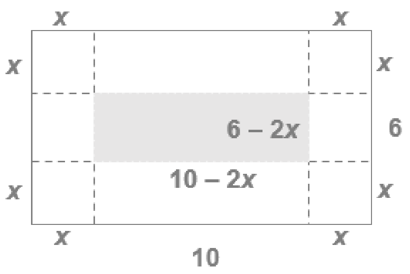
21 Area of a box:

**An open box is to be made from a sheet of cardboard 6 inches by 10 inches by cutting equal squares of side length "x" inches from each of the four corners and bending up the sides.**

**Express the area of the inner surface of the box, S, as a function of x**

a) $S = 4(15 - x)$
b) $S = 4(x^2 - 15)$
c) $S = 4(15 - x^2)$
d) $S = 4(x - 15)$
e) None of the above

Solution:




The area of the inner surface will be the whole original rectangle with sides 10-2*x* and 6-2*x* minus the four corners each with an area of $x^2$.

$$S = 10\cdot 6 - 4x^2 = 60 - 4x^2 = \underline{4\left(15 - x^2\right)}$$ c