Mathematics – Complex - Exercises - Solutions

Click on the exercise number or the 'Video' button to start a video.

01 Complex number, negative:

For the complex number -2 + 4i:

State the negative

a) -2 - 4i
b) -2 + 4i
c) 2 - 4i
d) 2 + 4i
e) None of the above

Solution:
The negative of a complex number z = x +iy is given by -z = -(x+iy) = -x-iy

$-[-2+4i] = \underline{2-4i}$ $\underline{\underline{c}}$

Video

02 Complex number, conjugate:

For the complex number -2 + 4i:

State the conjugate

a) 2 - 4i
b) -2 - 4i
c) 2 + 4i
d) -2 + 4i
e) None of the above

Solution:
The conjugate of a complex number x + iy is given by x- iy.

$\overline{-2+4i} = -2-4i$ $\underline{\underline{b}}$

Video

03 Complex equation:

Find the real values of x and y for which the equation is satisfied: $2(2x + y) + (6 - x)i = 2(xi - 2)$

a) x = -5, y = 2
b) x = -6, y = 2
c) x = 2, y = -5
d) x = 2, y = -6
e) None of the above

Solution:
Two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are equal
if the real parts are equal and the imaginary parts are equal, that is:
$x_1 = x_2$ and $y_1 = y_2$.

$$2(2x+y)+(6-x)i = 2(xi-2)$$
$$4x+2y+6i-xi = 2xi-4$$
$$(4x+2y)+(6-x)i = -4+2xi$$
$$\Downarrow$$
$$4x+2y=-4 \quad \wedge \quad 6-x=2x$$
$$\Downarrow$$
$$\underline{x=2} \quad \wedge \quad \underline{y=-6} \qquad\qquad \underline{\underline{d}}$$

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04 Complex number in polar form:

Express the complex number $\frac{1}{2}(\sqrt{3} - i)$ in polar form using the smallest non-negative value of its argument.

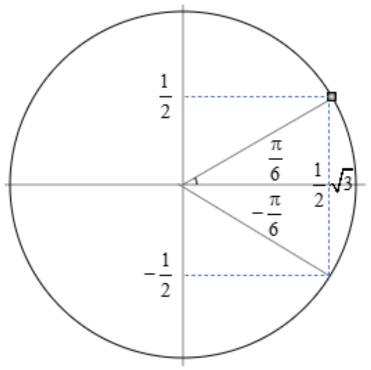
a) $\cos 30^\circ + i \sin 30^\circ$
b) $\cos 150^\circ + i \sin 150^\circ$
c) $\cos 210^\circ + i \sin 210^\circ$
d) $\cos 330^\circ + i \sin 330^\circ$
e) None of the above

Solution:

$$z = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$



$$\frac{1}{2}(\sqrt{3} - i) = \frac{1}{2}\sqrt{3} - \frac{1}{2}i = \cos 30^o - i\sin 30^o$$

$$= \cos 30^o + i(-\sin 30^o)$$

$$= \cos(360^o - 30^o) + i\sin(360^o - 30^o)$$

$$= \underline{\cos 330^o + i\sin 330^o} \qquad \underline{\underline{d}}$$

or

$$\frac{1}{2}(\sqrt{3} - i) = \frac{1}{2}\sqrt{3} - \frac{1}{2}i$$

$$r = \sqrt{\left(\frac{1}{2}\sqrt{3}\right)^2 + \left(-\frac{1}{2}\right)^2} = \sqrt{\frac{3}{4} + \frac{1}{4}} = \underline{1}$$

$$\theta = \tan^{-1}\left(\frac{-\frac{1}{2}}{\frac{1}{2}\sqrt{3}}\right) = \underline{330^o}$$

$$\frac{1}{2}(\sqrt{3} - i) = \frac{1}{2}\sqrt{3} - \frac{1}{2}i = \underline{\cos 330^o + i\sin 330^o}$$

Video

05 Complex number in rectangular form:

Express the complex number $3(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4})$ in rectangular form.

a) $-\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i$

b) $\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}i$

c) $\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$

d) $-\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}i$

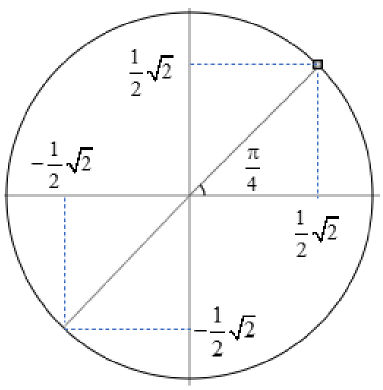
e) None of the above

Solution:

$$z = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$



$$3\left[\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right] = 3\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} = 3\cdot(-\frac{1}{2}\sqrt{2}) + i\cdot 3(-\frac{1}{2}\sqrt{2}) = -\frac{3\sqrt{2}}{2} - i\frac{3\sqrt{2}}{2}$$ a

Video

06 Complex number simplify:

Simplify $(-1-6i)^{-1}$ to the form $(a+bi)$.

a) $-\frac{1}{35}+\frac{6}{35}i$

b) $-\frac{1}{7}+\frac{6}{7}i$

c) $-\frac{1}{37}+\frac{6}{37}i$

d) $\frac{1}{7}+\frac{6}{7}i$

e) None of the above

Solution:

$$(-1-6i)^{-1}=-\frac{1}{1+6i}=-\frac{1\cdot(1-6i)}{(1+6i)(1-6i)}=-\frac{1-6i}{1^2-(6i)^2}=-\frac{1-6i}{7}=\underline{-\frac{1}{7}+\frac{6}{7}i}$$

b

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07 Complex number, multiplication:

Perform the multiplication of $(1+i)^3$:

Determine the smallest non-negative argument, in degrees, of the answer

a) 45°

b) 135°

c) 225°

d) 315°

e) None of the above

Solution:

For complex numbers we have:

$$z=x+iy=r(\cos\theta+i\sin\theta)=re^{i\theta}$$

$$z^n=(x+iy)^n=r^n(\cos n\theta+i\sin n\theta)=r^n e^{in\theta}$$

$$(1+i)^3=\left(\sqrt{1^2+1^2}\right)^3\left(e^{i\frac{\pi}{4}}\right)^3=\left(\sqrt{2}\right)^3 e^{i\frac{3\pi}{4}}=\underline{2\sqrt{2}e^{i\frac{3\pi}{4}}}$$

$$\theta=\frac{3\pi}{4}=\underline{135^o}$$

b

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08 Complex number, multiplication:

Perform the multiplication of $(1 + i)^3$:

Determine the modulus of the answer

a) 2
b) $\sqrt{2}$
c) $2\sqrt{2}$
d) $3\sqrt{3}$
e) None of the above

Solution:
For complex numbers we have:

$$z = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$$
$$z^n = (x+iy)^n = r^n(\cos n\theta + i\sin n\theta) = r^n e^{in\theta}$$

$$(1+i)^3 = \left(\sqrt{1^2+1^2}\right)^3 \left(e^{i\frac{\pi}{4}}\right)^3 = \left(\sqrt{2}\right)^3 e^{i\frac{3\pi}{4}} = \underline{2\sqrt{2}e^{i\frac{3\pi}{4}}}$$

$r = \underline{2\sqrt{2}}$ c

Video