Mathematics - Exercises - Linear Algebra - Solutions

Click on the exercise number to start a video.

01 Equations:

Solve the system of linear equations: 3x + 18y - 10 = 0 2x - 2y + 5 = 0

a) $x = -\frac{5}{3}$, $y = \frac{5}{6}$

b) $x = \frac{5}{3}$, $y = -\frac{5}{6}$

c) $x = -\frac{5}{6}$, $y = \frac{5}{3}$

d) $x = \frac{5}{6}$, $y = -\frac{5}{3}$

e) None of the above

Solution:

The general solution of a linear equation system of two unknowns is given by:

$$a_{11}x_1 + a_{12}x_2 = b_1$$
$$a_{21}x_1 + a_{22}x_2 = b_2$$

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} \qquad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

From this we have:

$$3x + 18y - 10 = 0$$
$$2x - 2y + 5 = 0$$
$$\Downarrow$$
$$3x + 18y = 10$$
$$2x - 2y = -5$$

$$x = \frac{\begin{vmatrix} 10 & 18 \\ -5 & -2 \end{vmatrix}}{\begin{vmatrix} 3 & 18 \\ 2 & -2 \end{vmatrix}} = \frac{10 \cdot (-2) - 18 \cdot (-5)}{3 \cdot (-2) - 18 \cdot 2} = \frac{70}{-42} = \underline{-\frac{5}{3}}$$

$$y = \frac{\begin{vmatrix} 3 & 10 \\ 2 & -5 \end{vmatrix}}{\begin{vmatrix} 3 & 18 \\ 2 & -2 \end{vmatrix}} = \frac{3 \cdot (-5) - 10 \cdot 2}{3 \cdot (-2) - 18 \cdot 2} = \frac{-35}{-42} = \underline{\frac{5}{6}}$$

$$(x, y) = \left(-\frac{5}{3}, \frac{5}{6}\right) \qquad \underline{\underline{a}}$$

02 Equation system:

For the system of linear equations:	4u - 3v + w + 5 = 0
	3u - v - 2w = 7
	u - 2v = w

Express the determinant of the system

a) $\begin{vmatrix} 4 & -3 & 1 \\ 3 & -1 & -2 \\ 1 & -2 & 0 \end{vmatrix}$

b) $\begin{vmatrix} 4 & -3 & 1 \\ 3 & -1 & -2 \\ 1 & -2 & -1 \end{vmatrix}$

c) $\begin{vmatrix} 4 & -3 & 1 \\ 3 & -1 & -2 \\ 1 & -2 & 1 \end{vmatrix}$

d) $\begin{vmatrix} 4 & -3 & 1 & 5 \\ 3 & -1 & -2 & -7 \\ 1 & -2 & -1 & 0 \end{vmatrix}$

e) None of the above

Solution:

The general solution of a linear equation system of three unknowns is given by:

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3
\end{aligned}
$$

$$
x_1 = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} \qquad
x_2 = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} \qquad
x_3 = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}
$$

From this we have:

$$
\begin{aligned}
&4u - 3v + w + 5 = 0 \\
&3u - v - 2w = 7 \\
&u - 2v = w \\
&\Updownarrow \\
&4u - 3v + w = -5 \\
&3u - v - 2w = 7 \\
&u - 2v - w = 0
\end{aligned}
$$

Determinant for the system (the determinant of the denominator)
is the coefficients in front of the unknown *u*, *v* and *w*.

$$
\begin{vmatrix} 4 & -3 & 1 \\ 3 & -1 & -2 \\ 1 & -2 & -1 \end{vmatrix}
$$

b

03 Equation system:

For the system of linear equations:	$4u - 3v + w + 5 = 0$
	$3u - v - 2w = 7$
	$u - 2v = w$

Express the numerator determinant for u

a) $\begin{vmatrix} -5 & -3 & 1 \\ 7 & -1 & -2 \\ 0 & -2 & 1 \end{vmatrix}$

b) $\begin{vmatrix} -5 & -3 & 1 \\ 7 & -1 & -2 \\ 0 & -2 & 0 \end{vmatrix}$

c) $\begin{vmatrix} -5 & -3 & 1 \\ 7 & -1 & -2 \\ 0 & -2 & -1 \end{vmatrix}$

d) $\begin{vmatrix} 4 & -3 & 1 \\ 3 & -1 & -2 \\ 1 & -2 & -1 \end{vmatrix}$

e) None of the above

Solution:

The general solution of a linear equation system of three unknowns is given by:

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3
\end{aligned}
$$

$$
x_1 = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} \qquad
x_2 = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} \qquad
x_3 = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}
$$

From this we have:

$$
\begin{aligned}
&4u - 3v + w + 5 = 0 \\
&3u - v - 2w = 7 \\
&u - 2v = w \\
&\Updownarrow \\
&4u - 3v + w = -5 \\
&3u - v - 2w = 7 \\
&u - 2v - w = 0
\end{aligned}
$$

Determinant for the numerator of *u*
is the the same as the coefficients in front of the unknown *u*, *v* and *w*,
except that the first column is replaced by the right side of the equation system.

$$
\begin{vmatrix} -5 & -3 & 1 \\ 7 & -1 & -2 \\ 0 & -2 & -1 \end{vmatrix} \qquad \underline{\underline{c}}
$$

04 Equation system:

For the system of linear equations:	$4u - 3v + w + 5 = 0$
	$3u - v - 2w = 7$
	$u - 2v = w$

Express the numerator determinant for v

a) $\begin{vmatrix} 4 & -3 & 1 \\ 3 & -1 & -2 \\ 1 & -2 & -1 \end{vmatrix}$

b) $\begin{vmatrix} 4 & 5 & 1 \\ 3 & 7 & -2 \\ 1 & 0 & -1 \end{vmatrix}$

c) $\begin{vmatrix} 4 & -5 & 1 \\ 3 & 7 & -2 \\ 1 & 0 & 1 \end{vmatrix}$

d) $\begin{vmatrix} 4 & -5 & 1 \\ 3 & 7 & -2 \\ 1 & 0 & -1 \end{vmatrix}$

e) None of the above

Solution:

The general solution of a linear equation system of three unknowns is given by:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}$$

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} \qquad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} \qquad x_3 = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

From this we have:

$$\begin{aligned} &4u - 3v + w + 5 = 0 \\ &3u - v - 2w = 7 \\ &u - 2v = w \\ &\Updownarrow \\ &4u - 3v + w = -5 \\ &3u - v - 2w = 7 \\ &u - 2v - w = 0 \end{aligned}$$

Determinant for the numerator of *v*
is the same as the coefficients in front of the unknown *u*, *v* and *w*,
except that the second column is replaced by the right side of the equation system.

$$\begin{vmatrix} 4 & -5 & 1 \\ 3 & 7 & -2 \\ 1 & 0 & -1 \end{vmatrix} \qquad \underline{\underline{d}}$$

05 Equation system:

For the system of linear equations:	**4u - 3v + w + 5 = 0**
	3u - v - 2w = 7
	u - 2v = w

Express the numerator determinant for w

a) $\begin{vmatrix} 4 & -3 & -5 \\ 3 & -1 & 7 \\ 1 & -2 & 0 \end{vmatrix}$

b) $\begin{vmatrix} 4 & -3 & -5 \\ 3 & -1 & 7 \\ 1 & -2 & -1 \end{vmatrix}$

c) $\begin{vmatrix} 4 & -3 & -5 \\ 3 & -1 & 7 \\ 1 & 0 & 0 \end{vmatrix}$

d) $\begin{vmatrix} 4 & -3 & 1 \\ 3 & -1 & -2 \\ 1 & -2 & -1 \end{vmatrix}$

e) None of the above

Solution:

The general solution of a linear equation system of three unknowns is given by:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}$$

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} \qquad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} \qquad x_3 = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

From this we have:

$$\begin{aligned} &4u - 3v + w + 5 = 0 \\ &3u - v - 2w = 7 \\ &u - 2v = w \\ &\Updownarrow \\ &4u - 3v + w = -5 \\ &3u - v - 2w = 7 \\ &u - 2v - w = 0 \end{aligned}$$

Determinant for the numerator of *w*
is the same as the coefficients in front of the unknown *u*, *v* and *w*,
except that the third column is replaced by the right side of the equation system.

$$\begin{vmatrix} 4 & -3 & -5 \\ 3 & -1 & 7 \\ 1 & -2 & 0 \end{vmatrix} \qquad \underline{\underline{a}}$$

06 Equation system:

For the system of linear equations:	**4u - 3v + w + 5 = 0**
	3u - v - 2w = 7
	u - 2v = w

Solve for u, v, and w

a) u = 2, v = 1, w = -3
b) u = 1, v = 2, w = -3
c) u = 2, v = -3, w = 1
d) u = 1, v = -3, w = 2
e) None of the above

Solution:

The general solution of a linear equation system of three unknowns is given by:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned}$$

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} \qquad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}} \qquad x_3 = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}}$$

From this we have:

$$
\begin{aligned}
&4u-3v+w+5=0\\
&3u-v-2w=7\\
&u-2v=w\\
&\Updownarrow\\
&4u-3v\ +\ \ w\ =-5\\
&3u-\ v\ -2w\ =\ \ 7\\
&\ u-2v\ -\ \ w\ =\ \ 0
\end{aligned}
$$

Determinant in the denominator equal for all unknown

$$
\begin{vmatrix}4 & -3 & 1\\ 3 & -1 & -2\\ 1 & -2 & -1\end{vmatrix} = 4\cdot\begin{vmatrix}-1 & -2\\ -2 & -1\end{vmatrix} - (-3)\cdot\begin{vmatrix}3 & -2\\ 1 & -1\end{vmatrix} + 1\cdot\begin{vmatrix}3 & -1\\ 1 & -2\end{vmatrix}
$$

$$
\begin{aligned}
&= 4\cdot\left[(-1)\cdot(-1)-(-2)\cdot(-2)\right]\\
&\quad +3\cdot\left[3\cdot(-1)-(-2)\cdot 1\right]\\
&\quad +1\cdot\left[3\cdot(-2)-(-1)\cdot 1\right]\\
&= 4\cdot\left[1-4\right]+3\cdot\left[-3+2\right]+1\cdot\left[-6+1\right]\\
&=-12-3-5\\
&=\underline{-20}
\end{aligned}
$$

$$
u=\frac{\begin{vmatrix}-5 & -3 & 1\\ 7 & -1 & -2\\ 0 & -2 & -1\end{vmatrix}}{\begin{vmatrix}4 & -3 & 1\\ 3 & -1 & -2\\ 1 & -2 & -1\end{vmatrix}}=\frac{-20}{-20}=\underline{\underline{1}}
\qquad
v=\frac{\begin{vmatrix}4 & -5 & 1\\ 3 & 7 & -2\\ 1 & 0 & -1\end{vmatrix}}{\begin{vmatrix}4 & -3 & 1\\ 3 & -1 & -2\\ 1 & -2 & -1\end{vmatrix}}=\frac{-40}{-20}=\underline{\underline{2}}
\qquad
w=\frac{\begin{vmatrix}4 & -3 & -5\\ 3 & -1 & 7\\ 1 & -2 & 0\end{vmatrix}}{\begin{vmatrix}4 & -3 & 1\\ 3 & -1 & -2\\ 1 & -2 & -1\end{vmatrix}}=\frac{60}{-20}=\underline{\underline{-3}}
$$

$(u,v,w)=(1,2-3)$ $\underline{\underline{b}}$