Mathematics – Trigonometry - Exercises - Solutions

Click on the exercise number to start a video.

01 Trigonometry, sin:

Given θ = 495 degrees, state the values for:

Sin θ

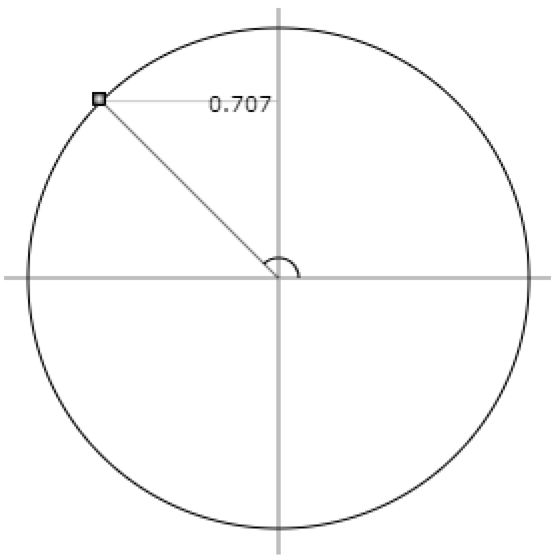
a) 0.866
b) - 0.707
c) 0.707
d) - 0.866
e) None of the above

Solution:

In the figure we have drawn the unit circle (radius equal 1) and the angle 135°.
Since 495° = 135° + 360°, the periphery point for each of the two angles 495° and 135° will be the same and therefore give the same value for the trigonometric functions.

Sinus (sin) of an angle is defined as the second-coordinate of the periphery point.

$\theta = 495^o \Rightarrow \sin\theta = \underline{0.707}$ c



02 Trigonometry, cos:

Given θ = 495 degrees, state the values for:

Cos θ

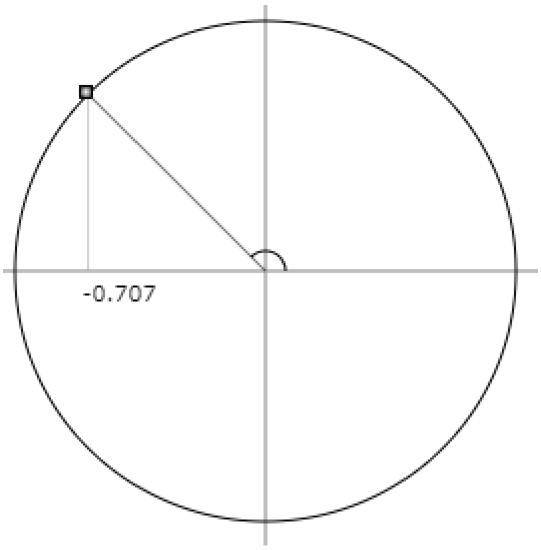
a) 0.866
b) - 0.866
c) - 0.707
d) 0.707
e) None of the above

Solution:

In the figure we have drawn the unit circle (radius equal 1) and the angle 135°.
Since 495° = 135° + 360°, the periphery point for each of the two angles 495° and 135° will be the same and therefore give the same value for the trigonometric functions.

Cosine (cos) of an angle is defined as the first-coordinate of the periphery point..

$\theta = 495^o \Rightarrow \cos\theta = \underline{-0.707}$ $\underline{\underline{c}}$



03 Trigonometry, angle

Given θ = 495 degrees, state the values for:

θ in radians

a) $\frac{5\pi}{4}$

b) $\frac{7\pi}{4}$

c) $\frac{9\pi}{4}$

d) $\frac{11\pi}{4}$

e) None of the above

Solution:

Conversion between degrees and radians:

$$\frac{\text{radians}}{\text{degrees}} = \frac{\pi}{180^o}$$

$$\Downarrow$$

$$\text{radians} = \text{degrees} \cdot \frac{\pi}{180^o} \qquad \text{degrees} = \text{radians} \cdot \frac{180^o}{\pi}$$

$$\theta = 495^o \Rightarrow \theta = 495^o \cdot \frac{\pi}{180^o} = \underline{\frac{11\pi}{4}}$$

$\underline{\underline{d}}$

04 Trigonometry, sin:

Given $\theta = \frac{7\pi}{6}$ radians, state the values for:

Sin θ

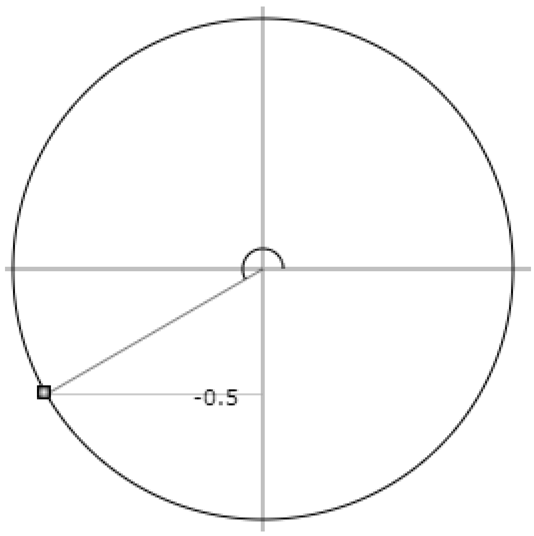
a) - 0.5
b) 0.5
c) 0.866
d) - 0.866
e) None of the above

Solution:

In the figure we have drawn the unit circle (radius equal 1) and the angle 7π/6 (= 210°).

Sinus (sin) of an angle is defined as the second-coordinate of the periphery point.

$$\theta = \frac{7\pi}{6} \Rightarrow \sin\theta = \underline{-0.5} \qquad \underline{\underline{a}}$$



05 Trigonometry, cos:

Given $\theta = \frac{7\pi}{6}$ radians, state the values for:

Cos θ

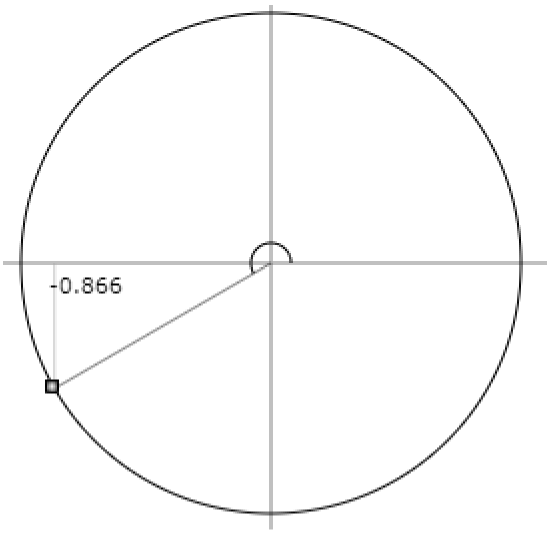
a) - 0.5
b) 0.5
c) - 0.866
d) 0.866
e) None of the above

Solution:

In the figure we have drawn the unit circle (radius equal 1) and the angle $7\pi/6$ (= 210°).

Cosine (cos) of an angle is defined as the first-coordinate of the periphery point.

$$\theta = \frac{7\pi}{6} \Rightarrow \cos\theta = \underline{-0.866} \qquad \underline{\underline{c}}$$



06 Trigonometry, angle:

Given $\theta = \frac{7\pi}{6}$ radians, state the values for:

θ in degrees

a) 150°
b) 210°
c) 390°
d) 420°
e) None of the above

Solution:

Conversion between degrees and radians:

$$\frac{\text{radians}}{\text{degrees}} = \frac{\pi}{180^o}$$
$$\Downarrow$$
$$\text{radians} = \text{degrees} \cdot \frac{\pi}{180^o} \qquad \text{degrees} = \text{radians} \cdot \frac{180^o}{\pi}$$

$$\theta = \frac{7\pi}{6} \Rightarrow \theta = \frac{7\pi}{6} \cdot \frac{180^o}{\pi} = \underline{210^o}$$

b

07 Trigonometry, angle in degrees:

Given Sin θ = 0 and Cos θ = - 1.0, state the values for:

θ in degrees

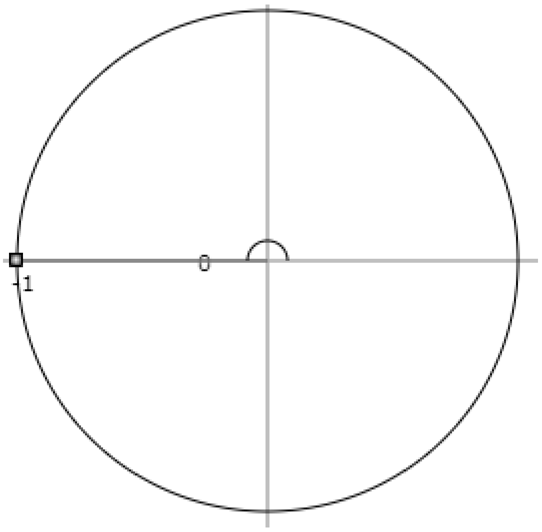
a) 0°
b) 90°
c) 180°
d) 270°
a) None of the above

Solution:

Sinus of an angle is equal zero when the angle is equal to 0°, 180° or 360°.
Cosine of an angle is equal -1 when the angle is equal 180°.
Sinus will be equal 0 and cosine equal -1 for the same angle when the angle is equal to 180°.

In the figure we have drawn the unit circle and the angle equal 180°.

$$\sin\theta = 0 \quad \wedge \quad \cos\theta = -1.0 \Rightarrow \underline{\theta = 180^o} \qquad \underline{\underline{c}}$$



08 Trigonometry, angle in radians:

Given Sin θ = 0 and Cos θ = - 1.0, state the values for:

θ in radians

a) $\frac{\pi}{2}$
b) $\frac{3\pi}{2}$
c) π
d) 2π
e) None of the above

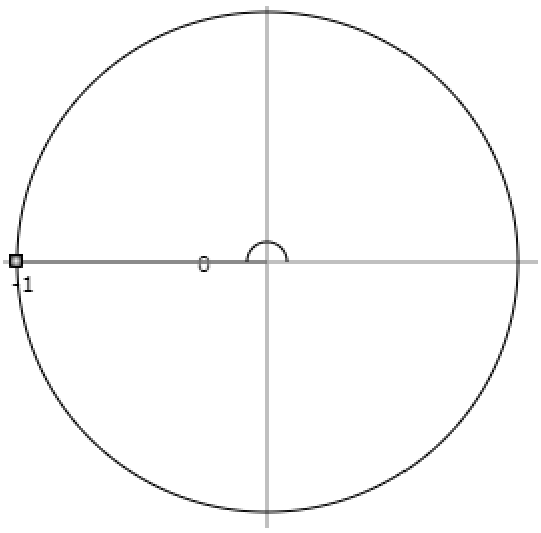
Solution:

Conversion between degrees and radians:

$$\frac{\text{radians}}{\text{degrees}} = \frac{\pi}{180^o}$$
$$\Downarrow$$

$$\text{radians} = \text{degrees} \cdot \frac{\pi}{180^o} \qquad \text{degrees} = \text{radians} \cdot \frac{180^o}{\pi}$$

$$\sin\theta = 0 \quad \wedge \quad \cos\theta = -1.0 \Rightarrow \theta = 180^o = 180^o \cdot \frac{\pi}{180^o} = \underline{\pi} \qquad \underline{\underline{c}}$$



09 Trigonometry, sin:

Given θ = -60 degrees, state the values for:

Sin θ

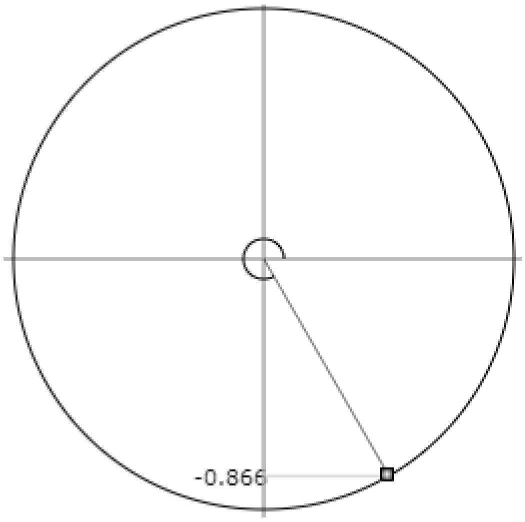
a) 0.866
b) 0.5
c) - 0.5
d) - 0.866
e) None of the above

Solution:

In the figure we have drawn the unit circle (radius equal 1) and the angle 300°.
Since -60° = 300° - 360°, the periphery point for each of the two angles -60° and 300° will be the same and therefore give the same value for the trigonometric functions.

Sinus (sin) of an angle is defined as the second-coordinate of the periphery point.

$\sin(-60^o) = \underline{-0.866}$ $\qquad\qquad \underline{\underline{d}}$



10 Trigonometry, cos:

Given θ = -60 degrees, state the values for:

Cos θ

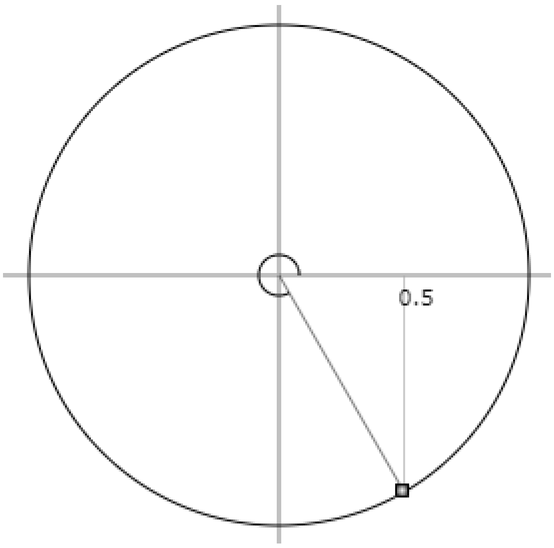
a) 0.5
b) 0.866
c) - 0.5
d) - 0.866
e) None of the above

Solution:

In the figure we have drawn the unit circle (radius equal 1) and the angle 300°.
Since (-60° = 300° - 360°, the periphery point for each of the two angles -60° and 300° will be the same and therefore give the same value for the trigonometric functions.

Cosine (cos) of an angle is defined as the second-coordinate of the periphery point.

$\cos(-60^o) = \underline{0.5}$ $\underline{\underline{a}}$



11 Trigonometry, angle:

Given θ = -60 degrees, state the values for:

θ in radians

a) $\frac{\pi}{3}$

b) $-\frac{\pi}{3}$

c) $-\frac{5\pi}{3}$

d) $-\frac{\pi}{6}$

e) None of the above

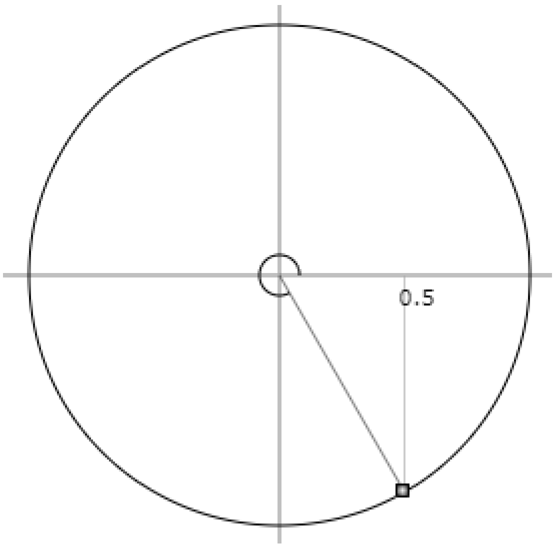
Solution:
Conversion between degrees and radians:

$$\frac{\text{radians}}{\text{degrees}} = \frac{\pi}{180^o}$$

$$\Downarrow$$

$$\text{radians} = \text{degrees} \cdot \frac{\pi}{180^o} \qquad \text{degrees} = \text{radians} \cdot \frac{180^o}{\pi}$$

$$\theta = -60^o \cdot \frac{\pi}{180^o} = \underline{-\frac{\pi}{3}} \qquad \underline{\underline{b}}$$



12 Geometry:

Solve the triangle given the three sides are: a = 5, b = 8, c = 11

a) Angle A = 24.6°, angle B = 89.0°, angle C = 66.4°
b) Angle A = 24.6°, angle B = 48.9°, angle C = 106.5°
c) Angle A = 34.6°, angle B = 55.4°, angle C = 90°
d) Angle A = 24.6°, angle B = 41.8°, angle C = 113.6°
e) None of the above

Solution:

Use the laws of sines and cosines to solve the triangle given the sides are 5, 8 and 11.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$$

$$A = \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right) = \cos^{-1}\left(\frac{5^2 - 8^2 - 11^2}{-2\cdot 8\cdot 11}\right) = 0.4297 = \underline{\underline{24.6^o}}$$

Use the sinus proportion:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\Downarrow$$

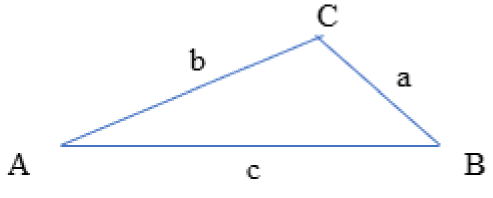
$$\sin B = \frac{b}{a}\sin A$$

$$B = \sin^{-1}\left(\frac{b}{a}\sin A\right) = \sin^{-1}\left(\frac{8}{5}\sin 24.62^o\right) = \underline{\underline{41.8^o}}$$

$$A + B + C = 180^o$$

$$\Downarrow$$

$$C = 180^o - A - B = 180^o - 24.6^o - 41.8^o = \underline{\underline{113.6^o}}$$ $\underline{\underline{d}}$



13 Airplane:

An airplane takes off and continues to rise at an angle of 8 degrees with the horizontal. By how many feet will it clear a horizontal wire 50 feet high at a distance of 500 feet from takeoff?

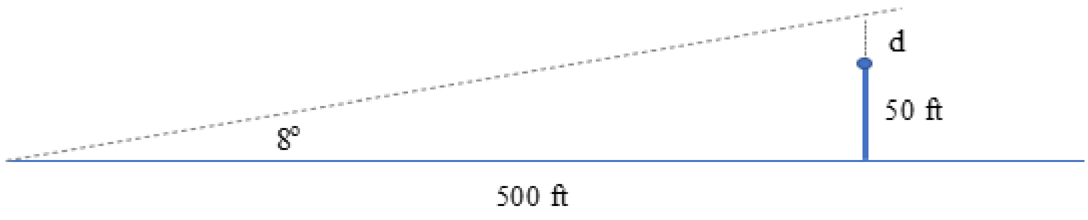
a) 10 feet
b) 20 feet
c) 30 feet
d) 40 feet
e) None of the above

Solution:

$$\tan(8^o) = \frac{50 \text{ ft} + d}{500 \text{ ft}}$$

$$\Downarrow$$

$$d = 500 \text{ ft} \cdot \tan(8^o) - 50 \text{ ft} = 70 \text{ ft} - 50 \text{ ft} = \underline{20 \text{ ft}}$$ b



14 Damped oscillation:

For the function $Y = -3e^{-2t}\sin(\pi t + \frac{\pi}{3})$:

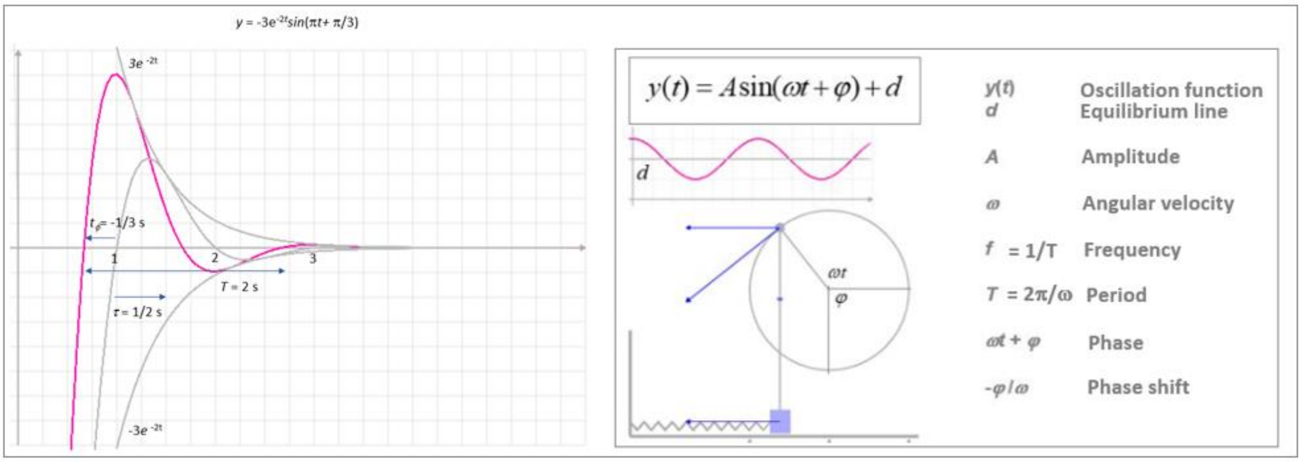
Calculate the period

a) 1 second
b) 2 seconds
c) 3 seconds
d) 4 seconds
e) None of the above

Solution:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi\ s^{-1}} = \underline{2\ s}$$

b



Click on the figure to start the simulation.

15 Damped oscillation:

For the function $Y = -3e^{-2t}\sin(\pi t + \frac{\pi}{3})$:

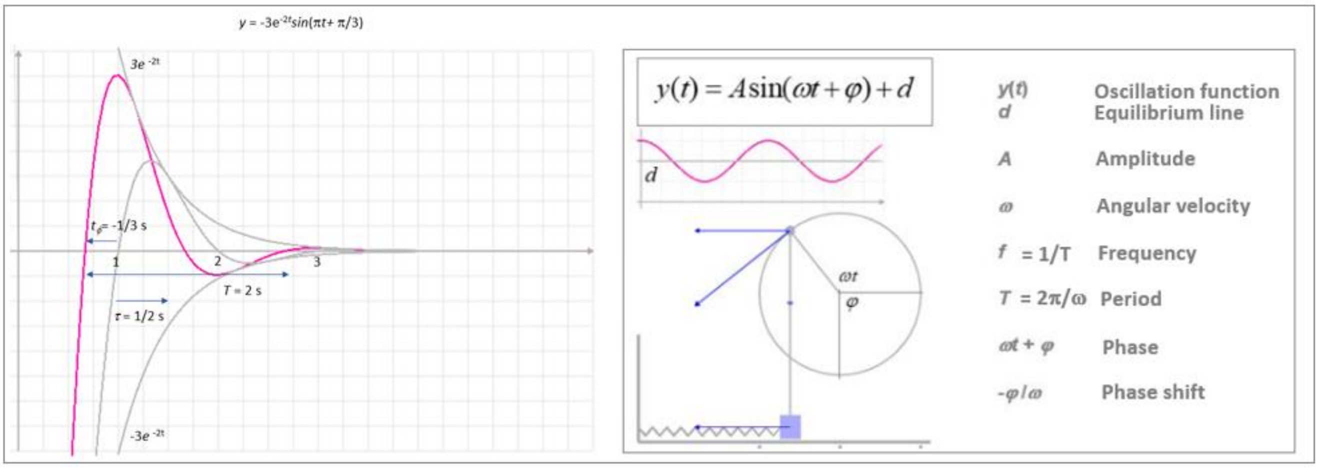
Calculate the frequency

a) $\frac{1}{2}$ cps
b) 1 cps
c) 2 cps
d) 3 cps
e) None of the above

Solution:

$$f = \frac{1}{T} = \frac{1}{2\ \mathrm{s}} = \frac{1}{2}\ \mathrm{s}^{-1} = \underline{\frac{1}{2}\ \mathrm{cps}}$$

a



Click on the figure to start the simulation.

16 Damped oscillation:

For the function $Y = -3e^{-2t}\sin(\pi t + \frac{\pi}{3})$:

Calculate the time (phase) shift

a) 3 seconds

b) $\frac{1}{3}$ second

c) 2 seconds

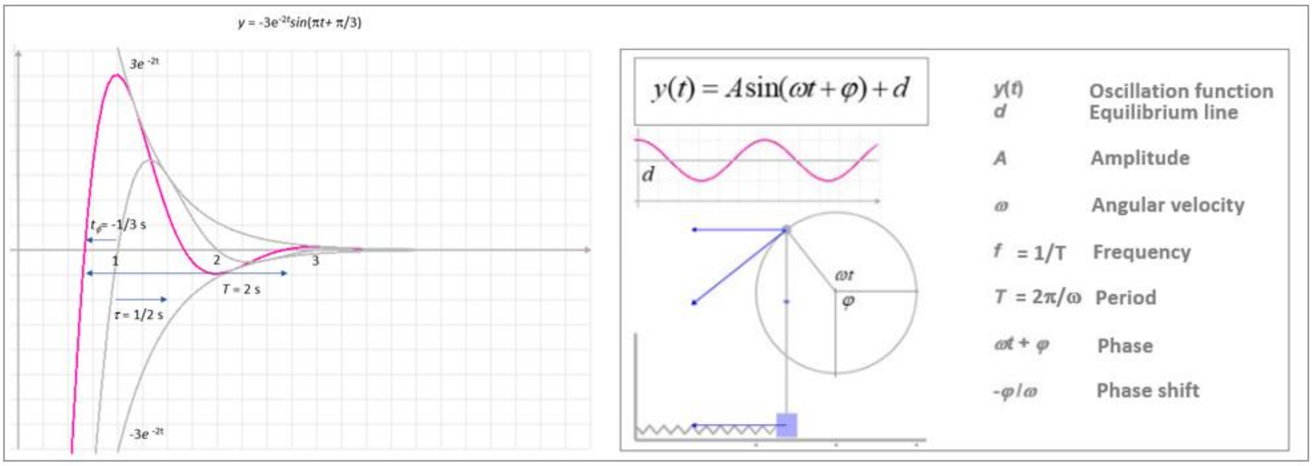
d) $-\frac{1}{3}$ second

e) None of the above

Solution:

$$\frac{-\varphi}{\omega} = \frac{-\left(\frac{\pi}{3}\right)}{\pi\ s^{-1}} = -\frac{1}{3}\ s$$

d



Click on the figure to start the simulation.

17 Damped oscillation:

For the function $Y = -3e^{-2t}\sin(\pi t + \frac{\pi}{3})$:

Calculate the time constant

a) $\frac{1}{2}$ second

b) 1 second

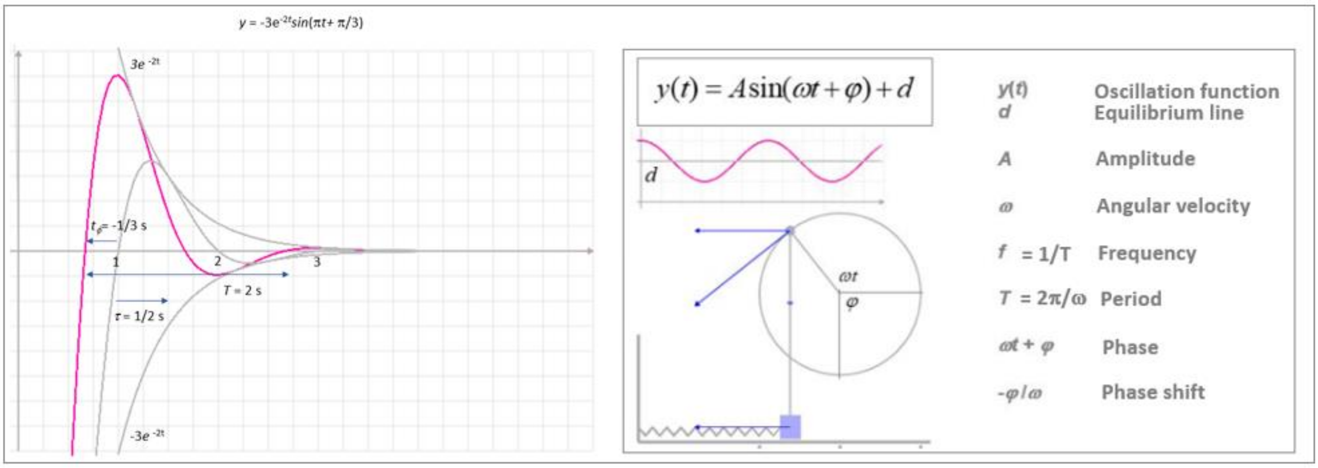
c) 2 seconds

d) 3 seconds

e) None of the above

Solution:

$$3e^{-2\ \mathrm{s}^{-1}\cdot t} = 3\frac{1}{e} = 3e^{-1} \Rightarrow -2\ \mathrm{s}^{-1}\cdot t = -1 \Rightarrow t = \tau = \frac{1}{2}\ \mathrm{s}$$ <u>a</u>



Click on the figure to start the simulation.

18 Damped oscillation:

For the function $D = 3e^{-t/3} \sin(\frac{\pi}{2}t - \frac{\pi}{4})$:

Calculate the period

a) $\frac{1}{4}$ second

b) $\frac{1}{3}$ seconds

c) 3 seconds

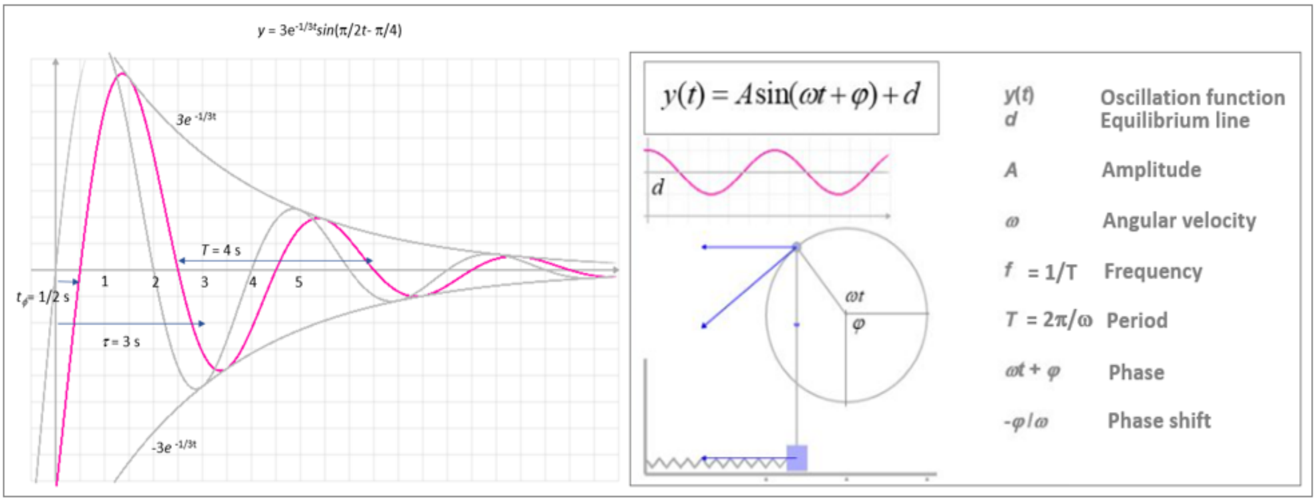
d) 4 seconds

e) None of the above

Solution:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\pi}{2}\ \mathrm{s}^{-1}} = \underline{4\ \mathrm{s}}$$

d



Click on the figure to start the simulation.

19 Damped oscillation:

For the function $D = 3e^{-t/3}\sin(\frac{\pi}{2}t - \frac{\pi}{4})$:

Calculate the frequency

a) $\frac{1}{4}$ cps

b) $\frac{1}{2}$ cps

c) 2 cps

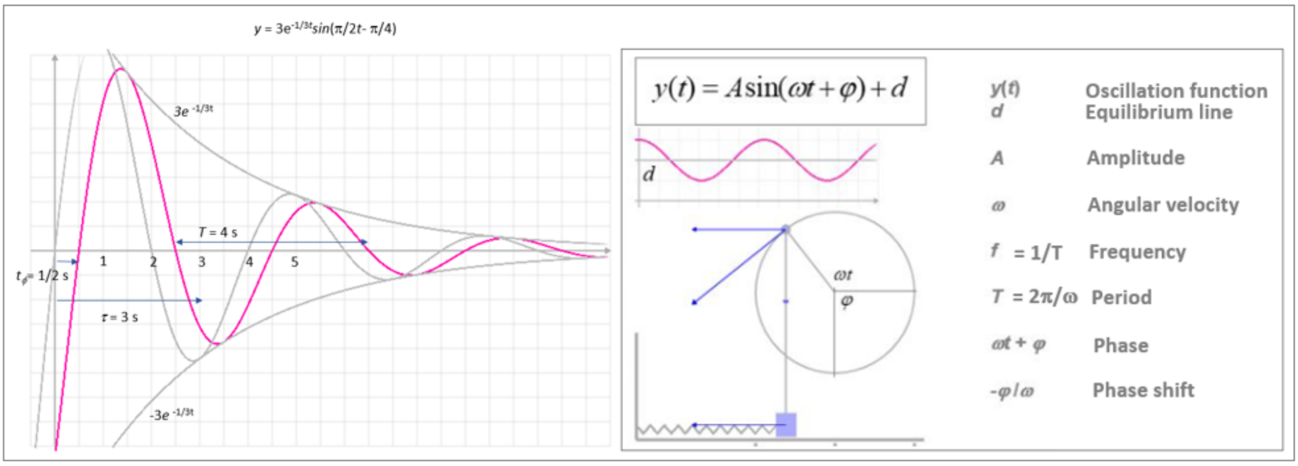
d) 4 cps

e) None of the above

Solution:

$$f = \frac{1}{T} = \frac{1}{4\text{ s}} = \frac{1}{4}\text{ s}^{-1} = \underline{\frac{1}{4}\text{ cps}}$$

$\underline{\underline{a}}$



Click on the figure to start the simulation.

20 Damped oscillation:

For the function $D = 3e^{-t/3}\sin(\frac{\pi}{2}t - \frac{\pi}{4})$:

Calculate the time (phase) shift

a) $-\frac{1}{2}$ second

b) $\frac{1}{2}$ second

c) 1 second

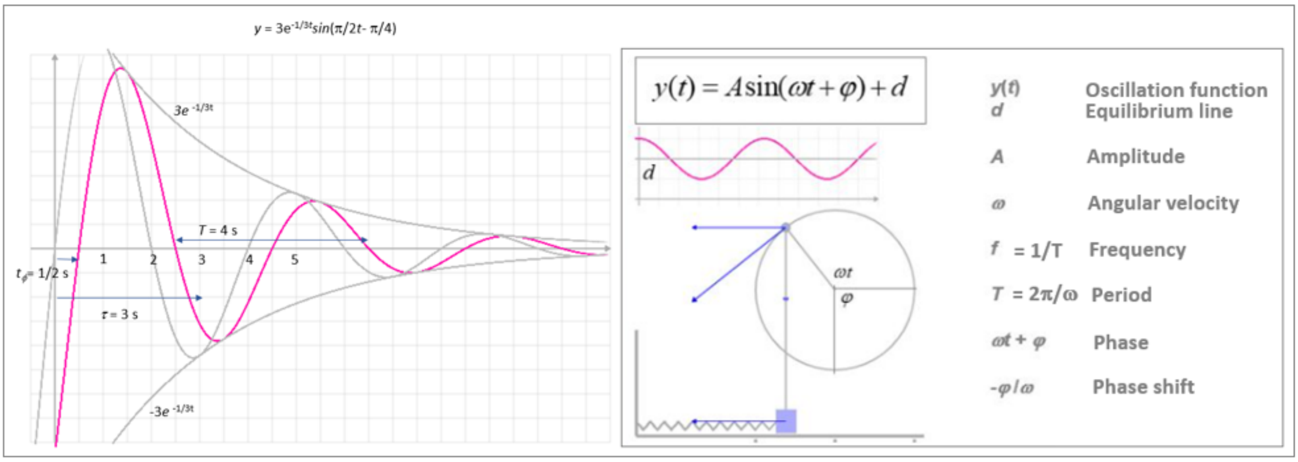
d) 2 seconds

e) None of the above

Solution:

$$\frac{-\varphi}{\omega} = \frac{-\left(-\frac{\pi}{4}\right)}{\frac{\pi}{2}\ \mathrm{s}^{-1}} = \underline{\frac{1}{2}\ \mathrm{s}}$$

b



Click on the figure to start the simulation.

21 Damped oscillation:

For the function $D = 3e^{-t/3}\sin(\frac{\pi}{2}t - \frac{\pi}{4})$:

Calculate the time constant

a) $\frac{1}{2}$ second

b) $\frac{1}{3}$ second

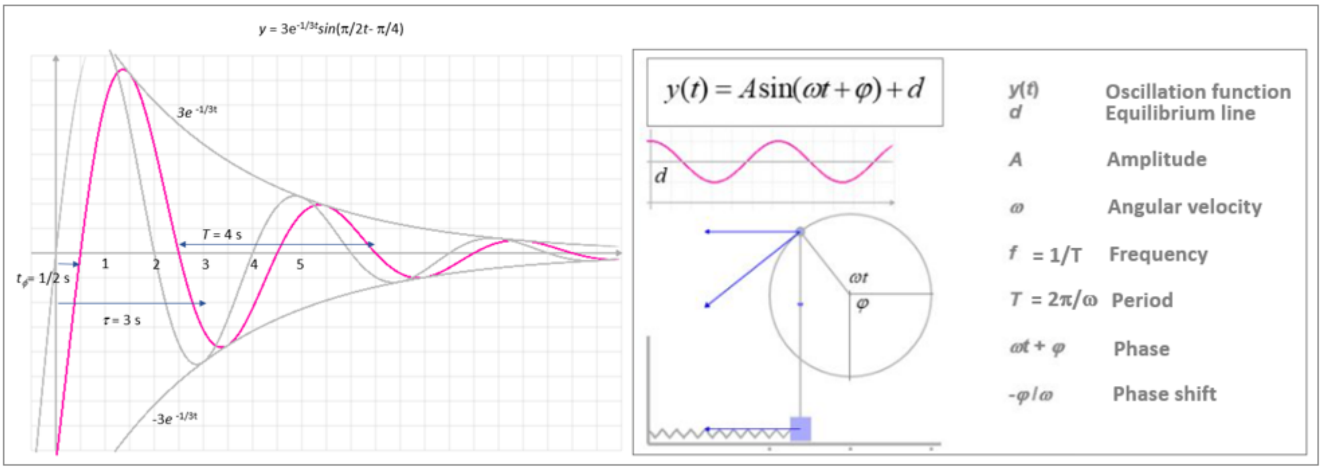
c) 2 seconds

d) 3 seconds

e) None of the above

Solution:

$$3e^{-\frac{1}{3}\text{s}^{-1}\cdot t} = 3\frac{1}{e} = 3e^{-1} \Rightarrow -\frac{1}{3}\text{ s}^{-1}\cdot t = -1 \Rightarrow t = \tau = 3\text{ s}$$ d



Click on the figure to start the simulation.