Mathematics - Complex - Exercises - Solutions

01 a) Express the complex number (-3-4i) in trigonometric form.

b) Express the complex number $3e^{i3\pi/4}$ in rectangular form.

c) Sketch the location of the complex number $8\,e^{i\pi}$ on the complex plane.

d) On the complex plane, sketch the graphical addition of the following numbers:

-1 + 2i and 3 – i

Solution:

We have three representations of complex numbers:

Rectangular form : $z = x + iy$

Trigonometric form : $z = r(\cos\theta + i\sin\theta)$

Exponential form : $z = re^{i\theta}$

Here we have $r = (x^2 + y^2)^{1/2}$ $\tan\theta = y/x$

a) Trigonometric form:

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{25} = \underline{5}$$

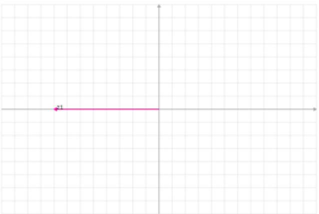
$$\tan\theta = \frac{y}{x} = \frac{-4}{-3} = \frac{4}{3} \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{4}{3}\right) = \underline{233.1^o}$$

$$-3 - 4i = \underline{\underline{5[\cos(233.1^o) + i\sin(233.1^o)]}}$$

b) Rectangular form:

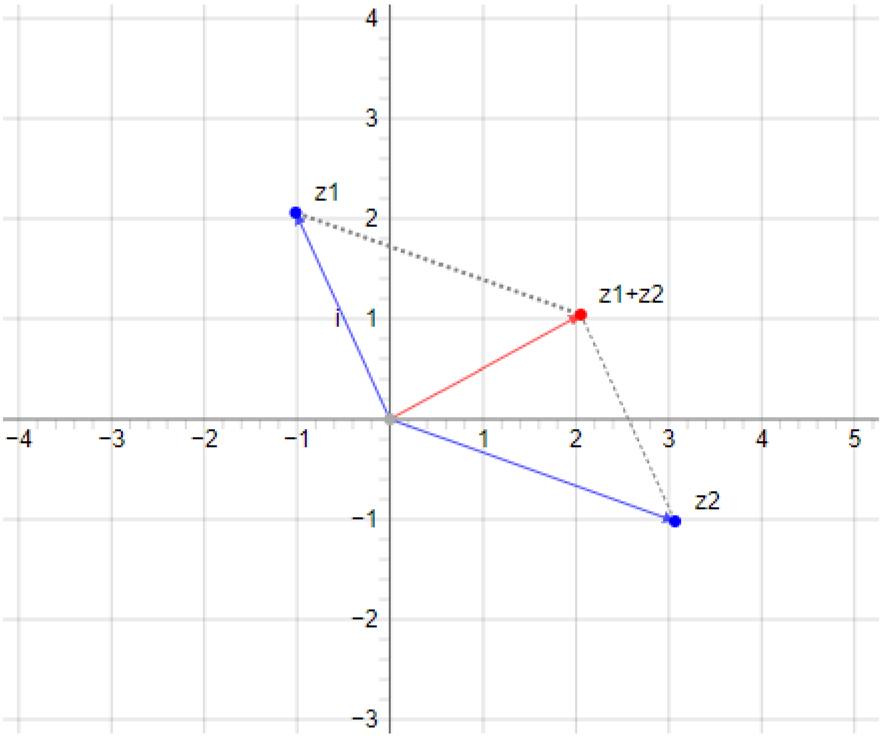
$$3e^{i\frac{3\pi}{4}} = 3[\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)] = 3(-0.7071 + i0.7071) = \underline{\underline{2.121 + 2.121i}}$$

c) The location of the complex number $8e^{i\pi}$:



d) The addition of the following two complex numbers: -1+2i and 3-i

(-1+2i) + (3-i) = (-1+3) + (2i-i) = 2+i



02 Perform the following mathematical operations:

a) (3-2i)/(2-3i)
b) $(3e^{i\pi})(-4e^{i\pi})$
c) $[2(\cos 3\pi/4 + i\sin 3\pi/4]^4$
d) $[16(\cos 3\pi/4 + i\sin 3\pi/4]^{1/4}$

Solution:

a) $$\frac{3-2i}{2-3i}=\frac{(3-2i)(2+3i)}{(2-3i)(2+3i)}=\frac{6-4i+9i-6i^2}{4-9i^2}=\frac{12+5i}{13}=\underline{\underline{\frac{12}{13}+\frac{5}{13}i}}$$

b) $$(3e^{i\pi})(-4e^{i\pi})=3\cdot(-4)e^{i(\pi+\pi)}=-12e^{i2\pi}=-12e^{i0}=\underline{\underline{-12}}$$

c) $$\left[2(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4})\right]^4=2^4\left[\cos(4\cdot\frac{3\pi}{4})+i\sin(4\cdot\frac{3\pi}{4})\right]$$
$$=16\left[\cos 3\pi+i\sin 3\pi\right]=16\left[-1+i\cdot 0\right]=\underline{\underline{-16}}$$

d) $$\left[16(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4})\right]^{\frac{1}{4}}=16^{1/4}\left[\cos(\frac{1}{4}\cdot\frac{3\pi}{4}+\frac{1}{4}\cdot 2\pi k)+i\sin(\frac{1}{4}\cdot\frac{3\pi}{4}+\frac{1}{4}\cdot 2k\pi k)\right]$$
$$=2\left[\cos\left(\frac{3\pi}{16}+\frac{2\pi k}{4}\right)+i\sin\left(\frac{3\pi}{16}+\frac{2\pi k}{4}\right)\right]$$
$$=2\left[\cos\left(\frac{3\pi}{16}+k\frac{\pi}{2}\right)+i\sin\left(\frac{3\pi}{16}+k\frac{\pi}{2}\right)\right]$$
$$=\underline{\underline{2e^{i\left(\frac{3\pi}{16}+k\frac{\pi}{2}\right)}}}\qquad k=0,1,2,3$$