

Mathematics – Linear Algebra - Exercises - Solutions

01 Given the simultaneous equations, set up the determinant solution for z (no need to solve):

$$6x - 2y - 3z = 2$$

$$2z + 7y = 0$$

$$-x + 2y - 4 = 3$$

Solution:

We order the three equations, with x in the first column, y in the second column and z in the third column on the left side of the equal sign.

The constants that don't contain x, y or z are moved to the right side of the equal sign.

Different methods can be used to solve these three equations.

Here we use [Cramer's rule](#).

$$6x - 2y - 3z = 2$$

$$0x + 7y + 2z = 0$$

$$-x + 2y + 0z = 7$$

$$\begin{bmatrix} 6 & -2 & -3 \\ 0 & 7 & 2 \\ -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 7 \end{bmatrix}$$

$$z = \frac{\begin{vmatrix} 6 & -2 & 2 \\ 0 & 7 & 0 \\ -1 & 2 & 7 \end{vmatrix}}{\begin{vmatrix} 6 & -2 & -3 \\ 0 & 7 & 2 \\ -1 & 2 & 0 \end{vmatrix}}$$

To find the value of z (and x and y) we use [the general rules for determinants](#).

02 Evaluate the determinants:

a) $\begin{vmatrix} 3 & 1 \\ -3 & 1 \end{vmatrix}$

b) $\begin{vmatrix} -1 & 3 & -2 \\ 2 & 1 & 2 \\ 0 & 3 & -2 \end{vmatrix}$

Solution:

a) The value of a 2x2 [determinant](#) is: $a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$

$$\begin{vmatrix} 3 & 1 \\ -3 & -1 \end{vmatrix} = 3 \cdot (-1) - (-3) \cdot 1 = -3 - (-3) = -3 + 3 = \underline{\underline{0}}$$

b) Use the general rules for compute the value of a [determinant](#):

We construct the determinant value by executing from one row or one column.

We try to find a row (column) with as many zeros as possible.

Here row number 3 or column number 1 contain one zero.

Here we start with row number 3.

The signs for each member in a determinant are as follows:

$$\begin{vmatrix} + & - & + & \dots \\ - & + & - & \dots \\ + & - & + & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix}$$

Then we have:

$$\begin{aligned} \begin{vmatrix} -1 & 3 & -2 \\ 2 & 1 & 2 \\ 0 & 3 & -2 \end{vmatrix} &= 0 \cdot \begin{vmatrix} 3 & -2 \\ 1 & 2 \end{vmatrix} - 3 \cdot \begin{vmatrix} -1 & -2 \\ 2 & 2 \end{vmatrix} + (-2) \cdot \begin{vmatrix} -1 & 3 \\ 2 & 1 \end{vmatrix} \\ &= 0[3 \cdot 2 - (-2) \cdot 1] - 3[(-1) \cdot 2 - (-2) \cdot 2] + (-2) \cdot [(-1) \cdot 1 - 3 \cdot 2] \\ &= 0 - 3 \cdot 2 - 2 \cdot (-7) \\ &= -6 + 14 \\ &= \underline{\underline{8}} \end{aligned}$$