Mathematics – Trigonometry - Exercises - Solutions

Click the exercise number to start a video.

01 Complete the following table of equivalencies:

Angle (degrees)	Angle (radians)	Sine of the angle	Cosine of the angle	Tangent of the angle
-135				
	6			
		-0.5		-0.577
Small	Small			

Solution:

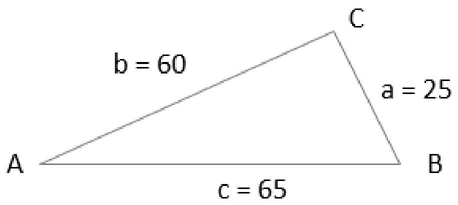
Angle (degrees)	Angle (radians)	Sine of the angle	Cosine of the angle	Tangent of the angle
-135	$-3\pi/4$	-0.7071	-0.7071	1
343.8	6	-0.279	0.960	-0.291
$330 + k\cdot 360$	$11\pi/6 + k\cdot 2\pi$	-0.5	0.866	-0.577
Small	Small	Angle (rad)	1	Angle (rad)

When using the calculator, remember the distinction between radians and degrees.

Notice that for small angles, Sine and Tangent is approximately equal to the angles (in radians), and Cosine is approximately equal to 1.

v (Deg)	v (Rad)	sin(v)	cos(v)	tan(v)
0	0	0	1	0
1	0.017	0.017	1	0.017
2	0.035	0.035	0.999	0.035
3	0.052	0.052	0.999	0.052
4	0.07	0.07	0.998	0.07
5	0.087	0.087	0.996	0.087
6	0.105	0.105	0.995	0.105
7	0.122	0.122	0.993	0.123
8	0.14	0.139	0.99	0.141
9	0.157	0.156	0.988	0.158
10	0.175	0.174	0.985	0.176
11	0.192	0.191	0.982	0.194
12	0.209	0.208	0.978	0.213
13	0.227	0.225	0.974	0.231
14	0.244	0.242	0.97	0.249
15	0.262	0.259	0.966	0.268

02 Use the laws of sines and cosines to solve the triangle given the sides are 25, 60 and 65 units long (show your work).



Solution:
Use the extended version of Pythagoras (the cosine theorem):

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\cos A = \frac{a^2 - b^2 - c^2}{-2bc}$$
$$A = \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right) = \cos^{-1}\left(\frac{25^2 - 60^2 - 65^2}{-2\cdot 60\cdot 65}\right) = \cos^{-1}\left(\frac{-7200}{-7800}\right) = \underline{\underline{22.62^o}}$$

Use the sinus proportion:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\Downarrow$$
$$\sin B = \frac{b}{a}\sin A$$
$$B = \sin^{-1}\left(\frac{b}{a}\sin A\right) = \sin^{-1}\left(\frac{60}{25}\sin 22.62^o\right) = \underline{\underline{67.38^o}}$$

$$A + B + C = 180^o$$
$$\Downarrow$$
$$C = 180^o - A - B = 180^o - 67.38^o - 22.62^o = \underline{\underline{90^o}}$$

One may wonder if the angle *C* is exactly or approximately 90 degrees.
We can control to see that $25^2 + 60^2 = 65^2$,
that is the numbers (25, 60, 65) form what we call a Pythagorean triple.
Therefore the angle C is exactly 90 degrees.
We could also short the triple (25, 60, 65) down to (5, 12, 13),
this way we see it even easier: $5^2 + 12^2 = 25 + 144 = 169 = 13^2$.
If we have discovered this triple in the beginning, we could of course used this instead of cosine theorem and sinus p roportion to compute the angles (sinA = a/c).

03 Given the function:

$$y = -1.5e^{-0.5t}\sin(2\pi t - \frac{\pi}{4})$$

a) Calculate the period *T*.
b) Calculate the frequency *f*.
c) Calculate the time shift (phase shift).
d) Calculate the envelope time constant.
e) Carefully sketch the function versus time.
f) On the sketch indicate the amplitude envelope, period, time shift (phase shift) and time constant.

Solution:

The general function of a damping oscillation is given by:

$$y = Ae^{-kt}\sin(\omega t + \varphi)$$

Where
|A| is the max amplitude
k is the damping constant
ω is the angular velocity
φ is the phase angle

Here we have:

$$y = -1.5e^{-0.5t}\sin(2\pi t - \frac{\pi}{4})$$

From this we have:

a) Period : $T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi s^{-1}} = \underline{\underline{1s}}$

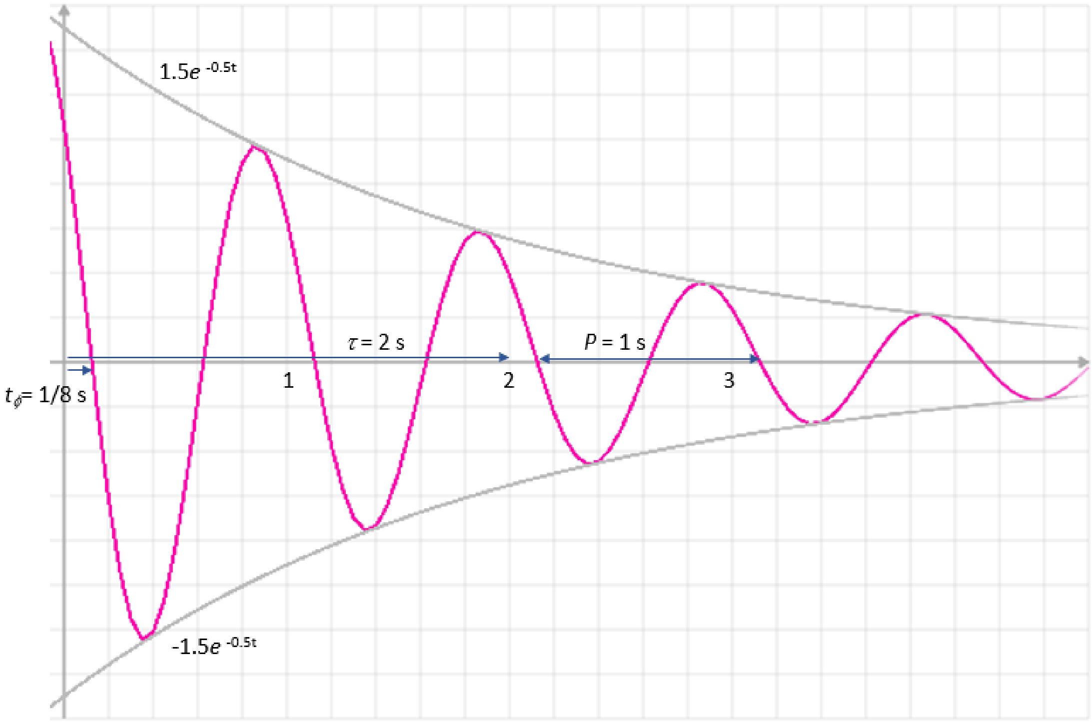
b) Frequency : $f = \frac{1}{T} = \frac{1}{1s} = \underline{\underline{1s^{-1}}}$

c) Time shift (phase shift) : $t_\phi = -\frac{\varphi}{\omega} = -\frac{-\frac{\pi}{4}}{2\pi s^{-1}} = \underline{\underline{\frac{1}{8}}}s$

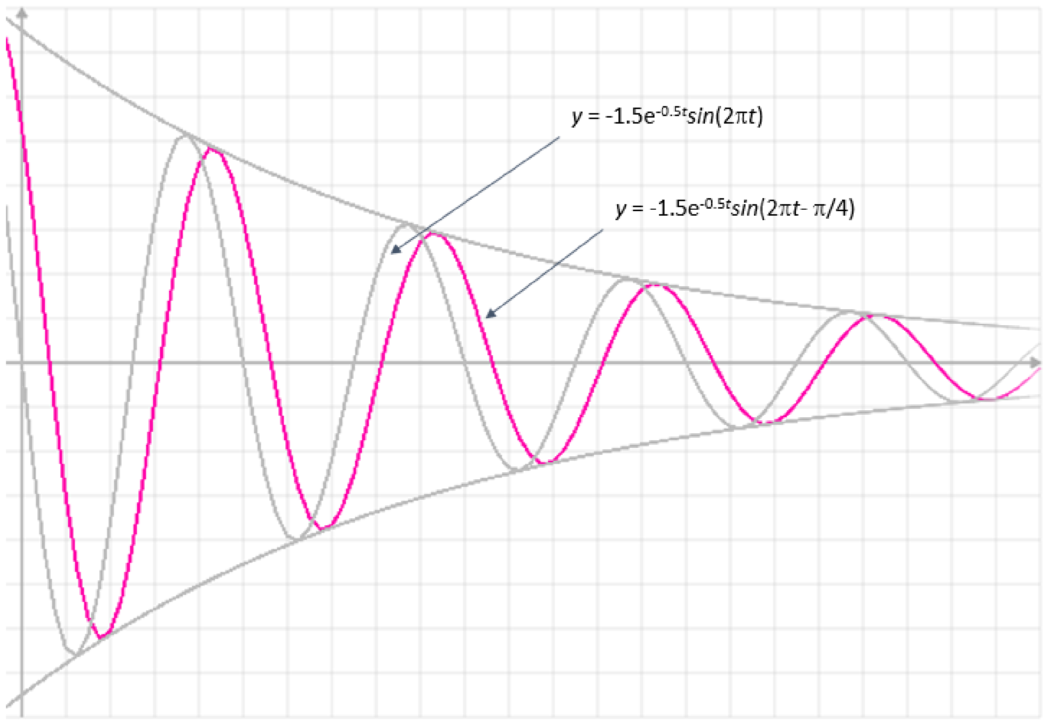
d) Envelope time constant : $Ae^{-kt} = A\cdot\frac{1}{e} = Ae^{-1} \Rightarrow -kt = -1 \Rightarrow t = \tau = \frac{1}{k} = \frac{1}{0.5s^{-1}} = \underline{\underline{2s}}$

The time constant τ is the time it will take until the amplitude Ae^{-kt} has decreased to 1/*e* of its original amplitude value *A*.

$y = -1.5e^{-0.5t}sin(2\pi t - \pi/4)$



Time shift (Phase shift)



04 Earth travels around the sun in an orbit that is almost circular.
Assume that the orbit is a circle with a radius of 93,000,000 miles.
Its angular and linear speed are used in designing solar-power facilities.

a) Assume that a year is 365 days, find the angle (in radians) formed by Earth's movement in one day.
b) Give the angular speed in radians per hour.
c) Find the linear speed of Earth in miles per hour.

Solution:

$$a)\quad \frac{2\pi}{365 day} = \underline{\underline{\frac{2\pi}{365} day^{-1}}}$$

$$b)\quad \frac{2\pi}{365} day^{-1} = \frac{2\pi}{365}(24 hour)^{-1} = \frac{2\pi}{365}\cdot\frac{1}{24} hour^{-1} = \underline{\underline{\frac{\pi}{4380} hour^{-1}}}$$

$$c)\quad v = r\omega = 93\cdot 10^6 mile \cdot \frac{\pi}{4380} hour^{-1} = 66000 \frac{mile}{hour} = \underline{\underline{6.6\cdot 10^4 \frac{mile}{hour}}}$$