FYSIKK FYS118-121-122 6/2-18

OPPSUMMERING

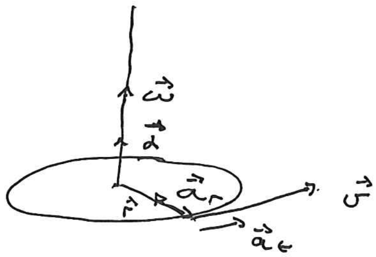
$$\theta = \frac{s}{r}$$

$$s = r\theta$$

$$v = r\omega$$

$$a_t = r\alpha$$

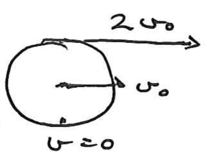
$$a_r = \frac{v^2}{r} = r\omega^2$$

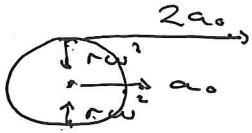


$$\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$$

$$\vec{a} = \underbrace{\vec{\alpha} \times \vec{r}}_{\vec{a}_t} + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r})}_{\vec{a}_r}$$

SYKLOIDE





ARBEID / ENERGI $\quad W = \int_C \vec{F} \cdot d\vec{r}$

KINETISK ENERGI $\quad E_k = \frac{1}{2} m v^2 \quad$ (TRANSLASJON)

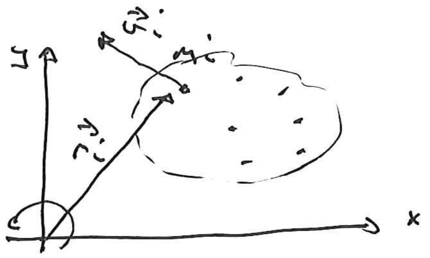
$$W_{\Sigma \vec{F}} = \Delta E_k$$

TRANSLASJON $K = E_k = \frac{1}{2}mv^2$

ROTASJON ?

HVOR MYE ENERGI MÅ VI TILFØRE ET SYSTEM
(STIVE SYSTEMER, AVSTANDEN MELLOM DE ENKELTE
DELENE I SYSTEMET ER KONSTANT).
FOR Å FÅ SYSTEMET OPP I EN GITT VINKELHASTIGHET ?

SYSTEM AV PUNKTPARTIKLER :



ROTERER OM z-AKSEN
(NORMALT UT AV PAPIRPLANET)
MED EN VINKELHASTIGHET ω.

KINETISK ENERGI FOR PARTIKKEL NR i (REN TRANSLASJON)

$$K_i = \frac{1}{2} m_i v_i^2$$

KINETISK ENERGI TIL HELE SYSTEMET :

$$K = \sum_i K_i = \sum_i \frac{1}{2} m_i v_i^2$$

$v = r\omega$

$$= \sum_i \frac{1}{2} m_i (r_i \omega_i)^2$$

$\omega_i = \omega$
FOR ALLE i
ALLE LIKE

$$= \sum_i \frac{1}{2} m_i r_i^2 \omega^2$$

$2 \cdot 4 + 3 \cdot 4 + 8 \cdot 4 + 10 \cdot 4$
$(2 + 3 + 8 + 10) \cdot 4$

$$= \frac{1}{2} \underbrace{\left(\sum_i m_i r_i^2 \right)}_{I} \omega^2$$

TREGHETSMOMENT

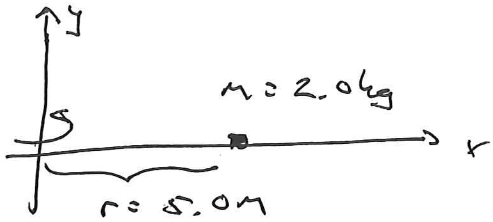
ROTASJON $\boxed{K = \frac{1}{2} I \omega^2 \quad \text{HVOR} \quad I = \sum_i m_i r_i^2}$

BENEVNING

TRANSLASJON $K = \frac{1}{2} m v^2$

$[I] = kg\,m^2$

EKS 1:



ROTASJON OM y-AKSEN

a) BESTEM TREGHETSMOMENTET OM y-AKSEN

b) HVOR MYE ENERGI MÅ TILFØRES FOR Å KOMME OPP I VINKELHASTIGHETEN $\omega = 2.0\,s^{-1}$

a) $I = \sum_i m_i r_i^2$

$= m r^2 = 2.0\,kg \cdot (5.0\,m)^2 = \underline{\underline{50\,kgm^2}}$

b) $K = \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot 50\,kgm^2 \cdot (2.0\,s^{-1})^2$

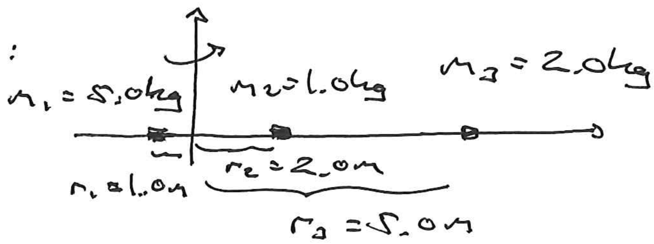
$N = \frac{kgm}{s^2}$

$= 100\,Nm$

$= \underline{\underline{100\,J}}$

—

EKS 2:



a) $I = \sum_i m_i r_i^2$

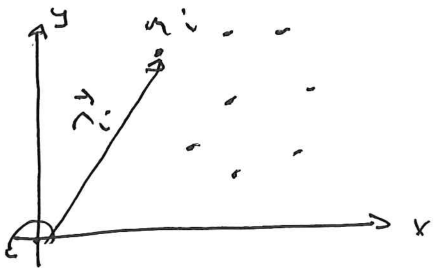
$= m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$

$= 5.0\,kg \cdot (1.0\,m)^2 + 1.0\,kg \cdot (2.0\,m)^2 + 2.0\,kg \cdot (5.0\,m)^2$

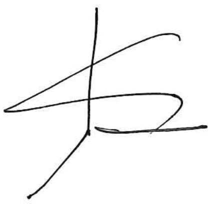
$= \underline{\underline{59\,kgm^2}}$

b) $K = \frac{1}{2} I \omega^2 = \frac{1}{2} \cdot 59\,kg\,m^2 \cdot (2.0\,s^{-1})^2 = \underline{\underline{118\,J}}$

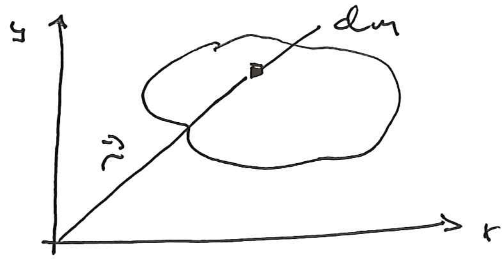
PUNKTPARTIKLER:



$$\boxed{I = \sum_i m_i r_i^2}$$

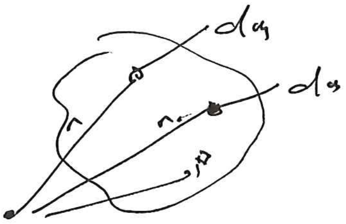


KONTINUERLIG MASSEFORDELING



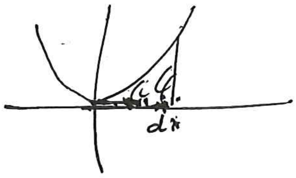
$$I = \int dm \cdot r^2$$

$$\boxed{I = \int r^2 \, dm}$$



$$\int \ldots dx$$

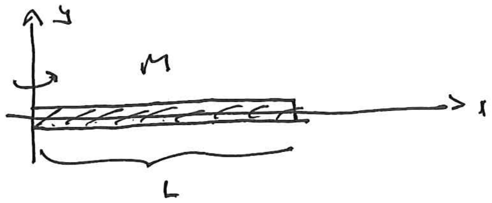
$$\int x^2 \, dx$$



$$\int x^2 \, dx$$

EKS1: VI HAR EN LANG, TYNN STAV MED LENGDE L OG MASSE M.

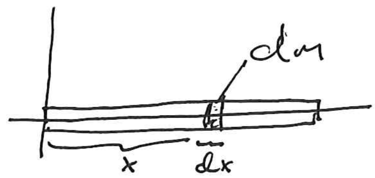
BESTEM STAVENS TREGHETSMOMENT OM EN AKSE NORMALT PÅ STAVEN I STAVENS ENE ENDEPUNKT.



TREGHETSMOMENT:

$$I = \int r^2 \, dm \,,$$

ALT LANGS X-AKSEN:



ρ MASSEN PR LENGDE-ENHET

$\rho = \frac{M}{L} \quad \frac{kg}{m}$

$M = \rho \cdot L$

$$I = \int x^2 \, dm$$

$$= \int x^2 \underbrace{\rho \, dx}_{dm}$$

$$= \rho \int_{x=0}^{x=L} x^2 \, dx$$

$$= \rho \left[\frac{1}{3} x^3 \right] \Bigg|_{x=0}^{x=L}$$

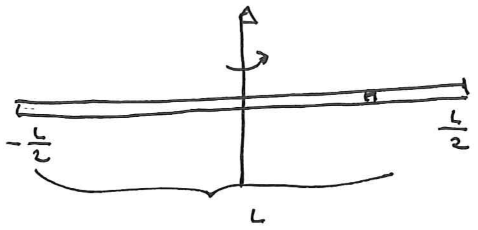
$$= \rho \cdot \frac{1}{3} L^3$$

$$= \frac{M}{\cancel{L}} \cdot \frac{1}{3} L^{\cancel{3}\,2}$$

$$= \underline{\underline{\frac{1}{3} M L^2}}$$

$$K = \frac{1}{2} I \omega^2$$

EKS 2:



ROTASJON OM EN AKSE MIDT PÅ STAVEN

$I = ?$

$$I = \int r^2 \, dm$$

$$= \int x^2 \, dm \quad \text{LANGS X-AKSEN}$$

$$= \int x^2 \rho \, dx \quad \rho = \text{MASSE PR LENGDE-ENHET}$$

$$= \rho \int_{x=-\frac{L}{2}}^{x=\frac{L}{2}} x^2 \, dx$$

$$= \rho \left[\frac{1}{3} x^3\right]\Big|_{x=-\frac{L}{2}}^{x=\frac{L}{2}}$$

$$= \rho \left[\frac{1}{3}\left(\frac{L}{2}\right)^3 - \frac{1}{3}\left(-\frac{L}{2}\right)^3\right]$$

$$= \frac{1}{12} \rho L^3$$

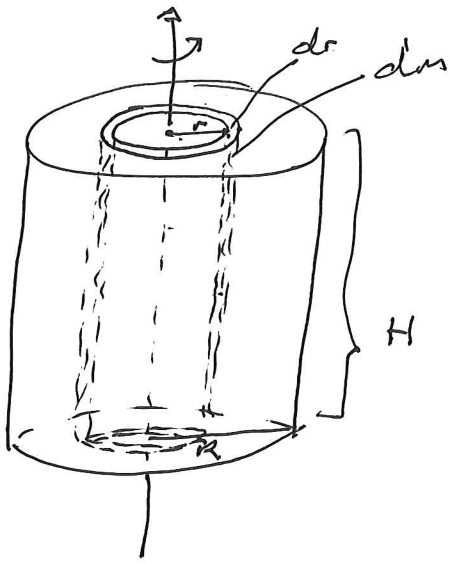
$$= \frac{1}{12} \frac{M}{L} L^3$$

$$= \underline{\underline{\frac{1}{12} M L^2}}$$

4 GANGER LETTERE ENN I EKS 1.

SENERE SKAL VI SE AT DET FINNES EN ENKEL OVERGANG MELLOM EKS 1 OG EKS 2.
(PARALLELLAKSETEOREMET)

EKS 3: ROTASJON AV EN MASSIV SYLINDER OM SENTERAKSEN.



MASSE M
HØYDE H
RADIUS R

$$I = \int r^2 \, dm$$

$$= \int r^2 \underbrace{\rho \, dV}_{dm}$$

HJELPEVARIABEL

$\rho = \frac{\text{MASSE}}{\text{VOLUM}}$

MASSE = $\rho \cdot$ VOLUM

~~MASSEN~~ (IN VOLUMET (INFINITESIMALT) AV SYLINDERSKALLET MED RADIUS r, TYKKELSE dr OG HØYDE H.

$$= \rho \int r^2 \, dV$$

$$= \rho \int r^2 \underbrace{2\pi r}_{\text{LENGDE OMKRETS}} \cdot \underbrace{H}_{\text{HØYDEN}} \cdot \underbrace{dr}_{\text{TYKKELSEN}}$$

$$= \rho \, 2\pi H \int_{r=0}^{r=R} r^3 \, dr$$

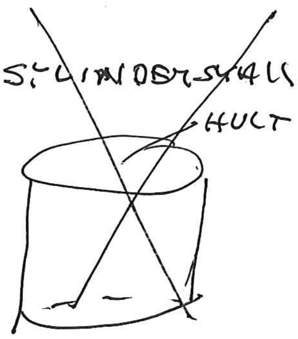
$$= \rho \, 2\pi H \left[\frac{1}{4} r^4 \right]_{r=0}^{r=R}$$

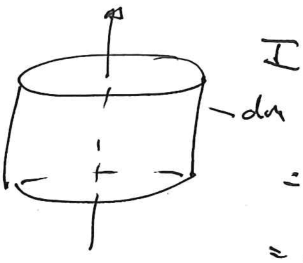
$$= \rho \, 2\pi H \, \frac{1}{4} \cdot R^4$$

$$= \frac{M}{\cancel{\pi} \cancel{R^2} \cancel{H}} \cdot \cancel{2\pi} \, \cancel{H} \cdot \frac{1}{\cancel{4}_2} R^{\cancel{4}\,2}$$

$$= \underline{\underline{\frac{1}{2} M R^2}}$$

SYLINDERSKALL HULT





$$I = \int r^2 \, dm$$

$$= \int R^2 \, dm$$

$$= R^2 \int dm$$

$$= R^2 M = \underline{\underline{M R^2}}$$