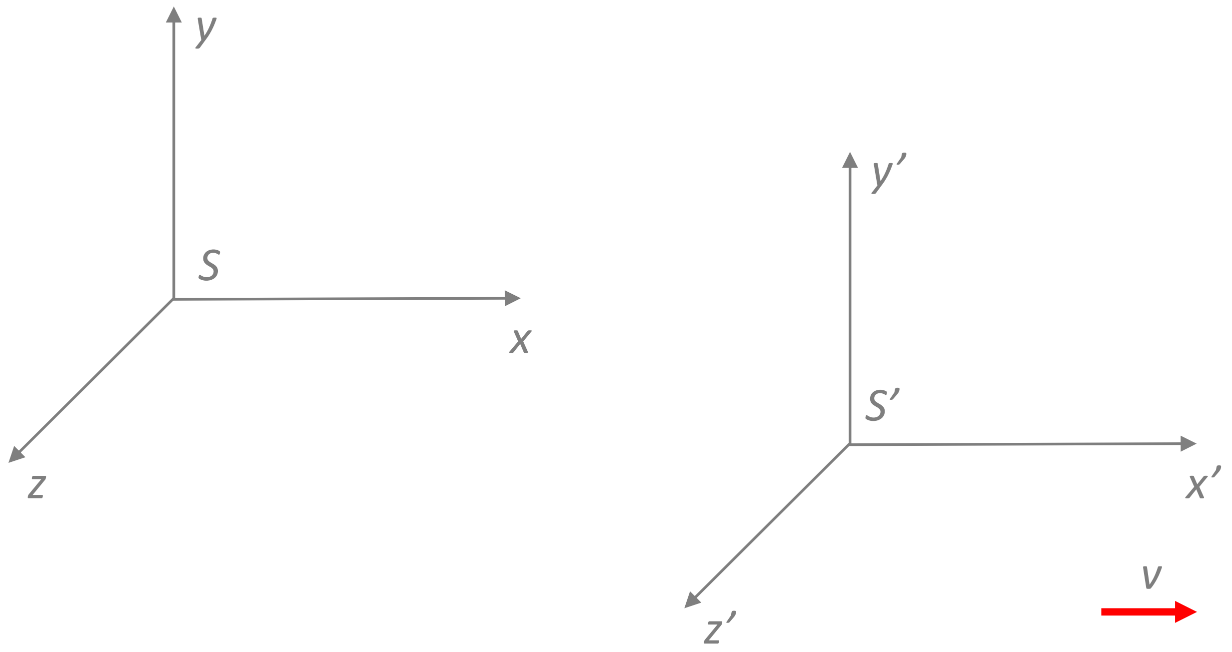
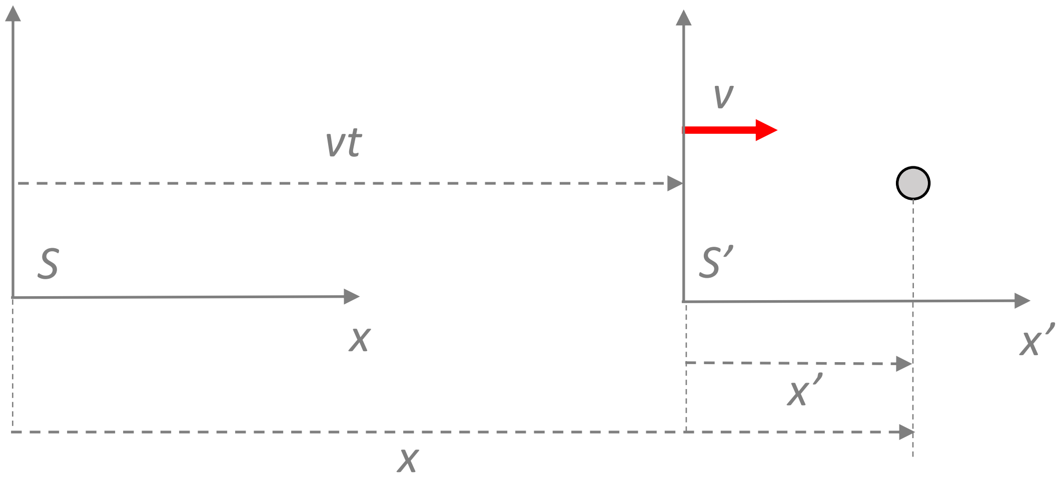
Galilean Transformation



$$x' = x - vt$$

Galilean Transformation



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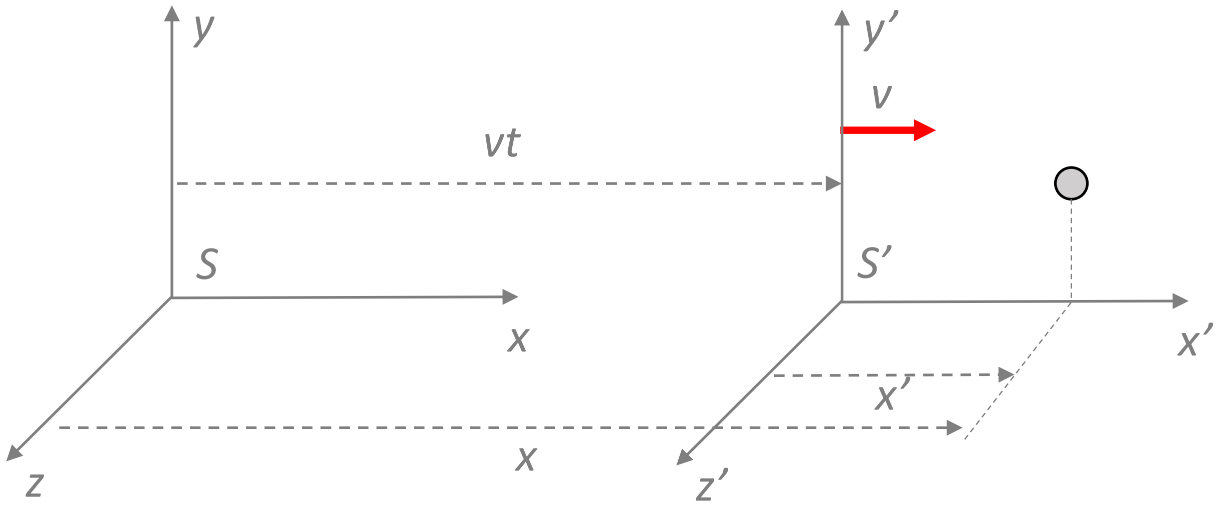
Galilean Transformation

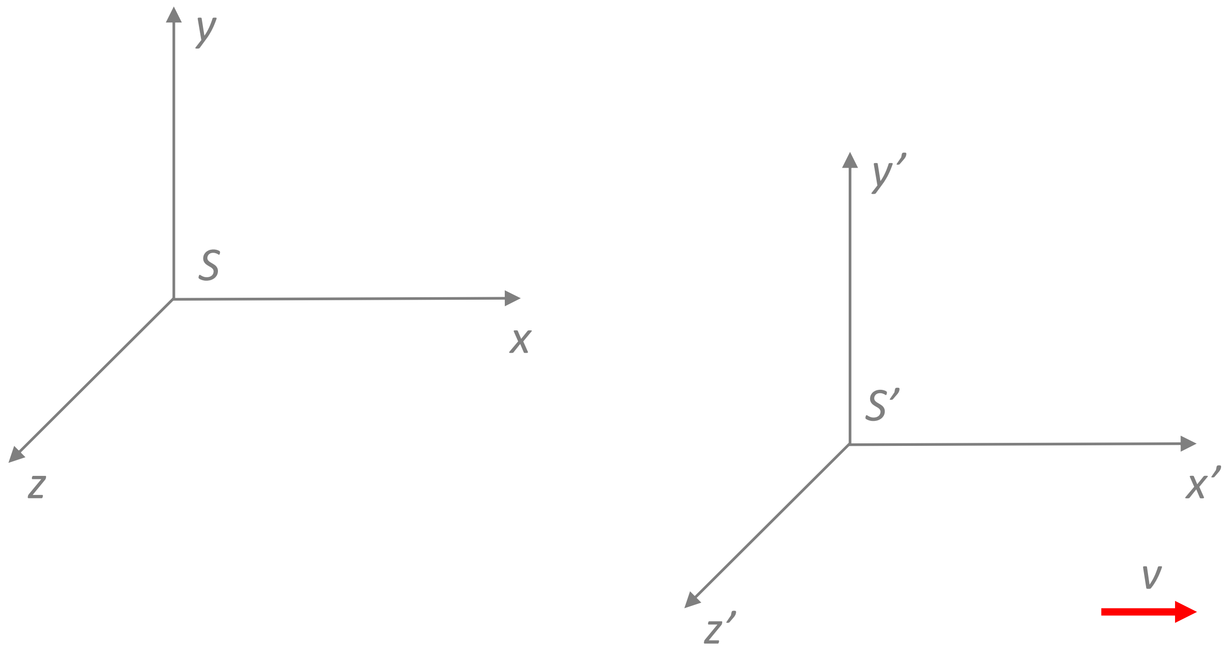


$$x' = x - vt$$

$$y' = y$$

$$z' = z$$

Galilean Transformation



$$x' = x - vt$$

$$y' = y$$

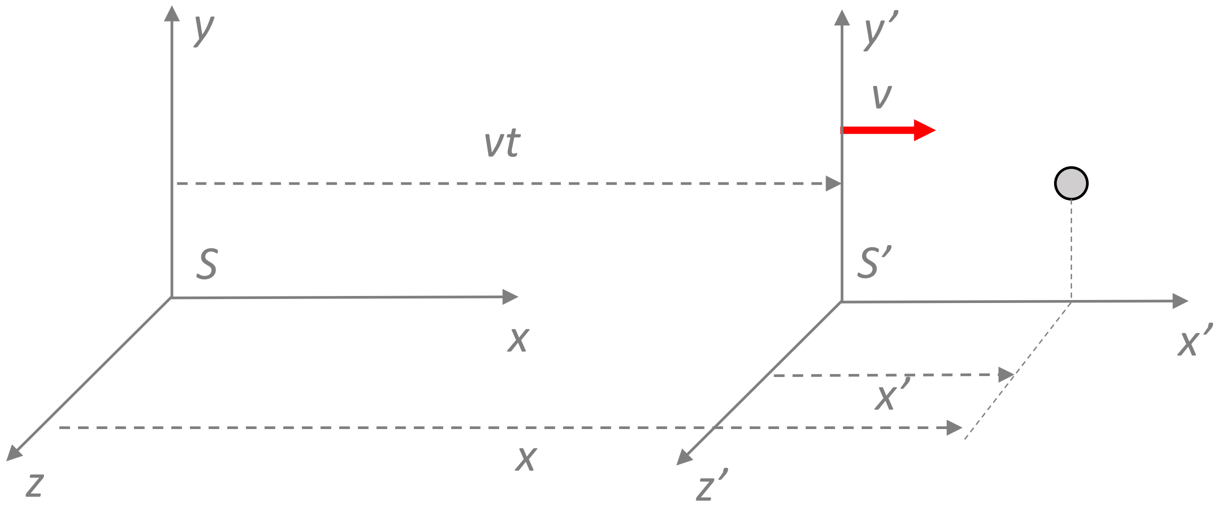
$$z' = z$$

$$V_x' = \frac{dx'}{dt} = v_x - v$$

$$V_y' = \frac{dy'}{dt} = v_y$$

$$V_z' = \frac{dz'}{dt} = v_z$$

Galilean Transformation



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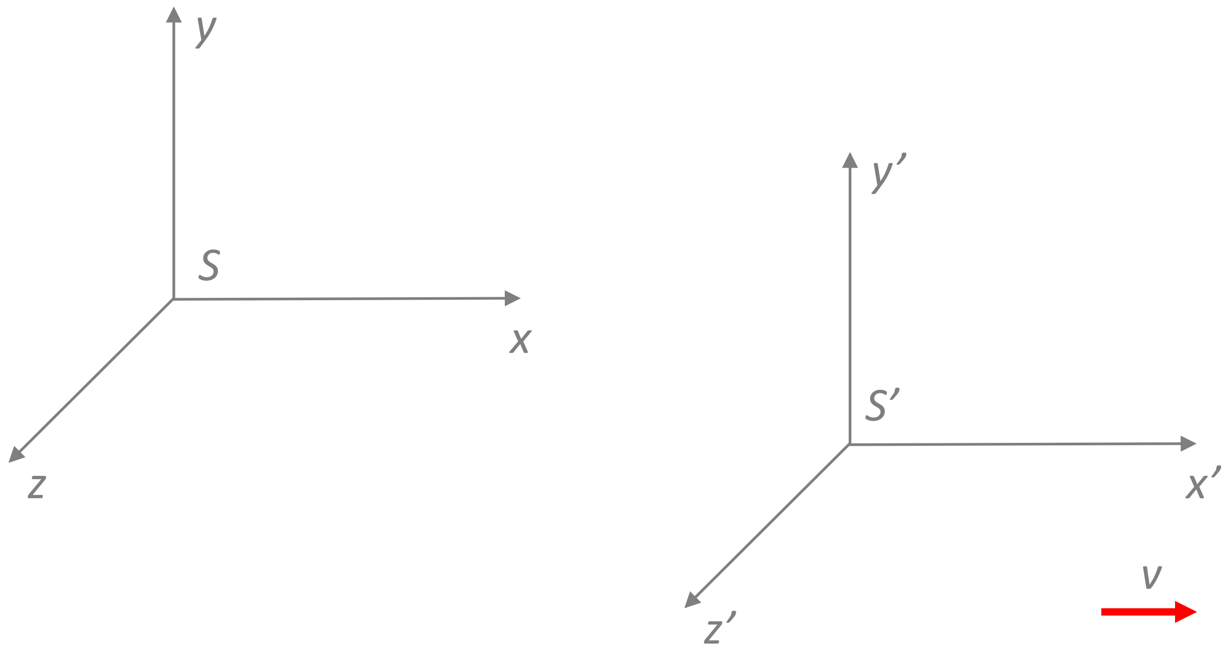
$$V_z' = \frac{dz'}{dt} = v_z$$

The Special Theory of Relativity
Postulates

1. The laws of physics may be expressed in equations having the same form in all frames of reference moving at constant velocity.

2. The speed of light in free space has the same value for all observers, regardless of their state of motion.

Lorentz Transformation



$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$

Lorentz Transformation

$$x' = \gamma(x - vt)$$
$$x = \gamma(x' + vt')$$

1. postulate of relativity

$$\Downarrow$$

$$x = \gamma(k(x - vt) + vt') = \gamma^2(x - vt) + \lambda vt' = \gamma^2 x - \gamma^2 vt + \gamma vt'$$

$$\Downarrow$$

$$t' = \gamma t + \frac{1 - \gamma^2}{\gamma v} x$$

Lorentz Transformation

$$x' = \gamma(x - vt)$$
$$x \;\; = \gamma(x' + vt')$$

1. postulate of relativity

$$t' = \gamma t + \frac{1-\gamma^2}{\gamma v}x$$

$$x \;\; = ct$$
$$x' = ct'$$

2. postulate of relativity

$$\gamma(x - vt) = c\left[\gamma t + \frac{1-\gamma^2}{\gamma v}x\right] = c\gamma t + \frac{1-\gamma^2}{\gamma v}cx$$

$$x = \frac{c\gamma t + v\gamma t}{\gamma - \frac{1-\gamma^2}{\gamma v}c} = ct\frac{\gamma + \frac{v}{c}\gamma}{\gamma - \frac{1-\gamma^2}{\gamma v}c} = ct\frac{1+\frac{v}{c}}{1-\left[\frac{1}{\gamma^2}-1\right]\frac{c}{v}} \overset{!}{=} ct$$

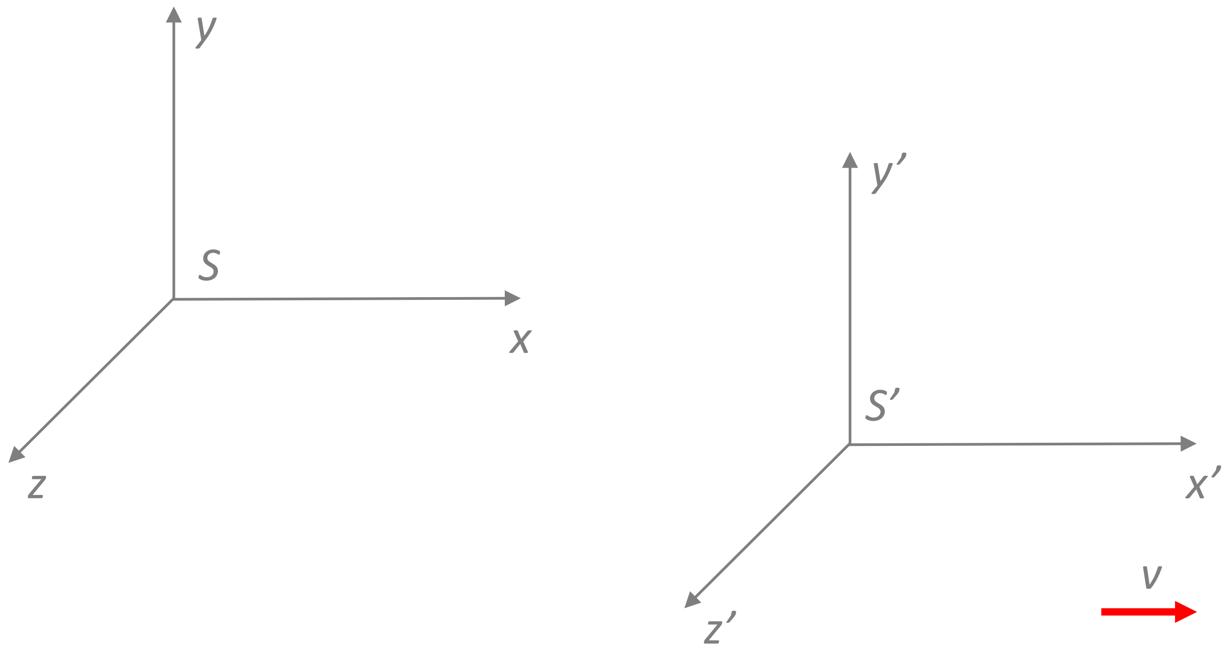
$$\Downarrow$$

$$\frac{1+\frac{v}{c}}{1-\left[\frac{1}{\gamma^2}-1\right]\frac{c}{v}} = 1$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \beta = \frac{v}{c}$$

Lorentz Transformation



$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

$$x = \gamma(x' + vt')$$

$$y = y'$$

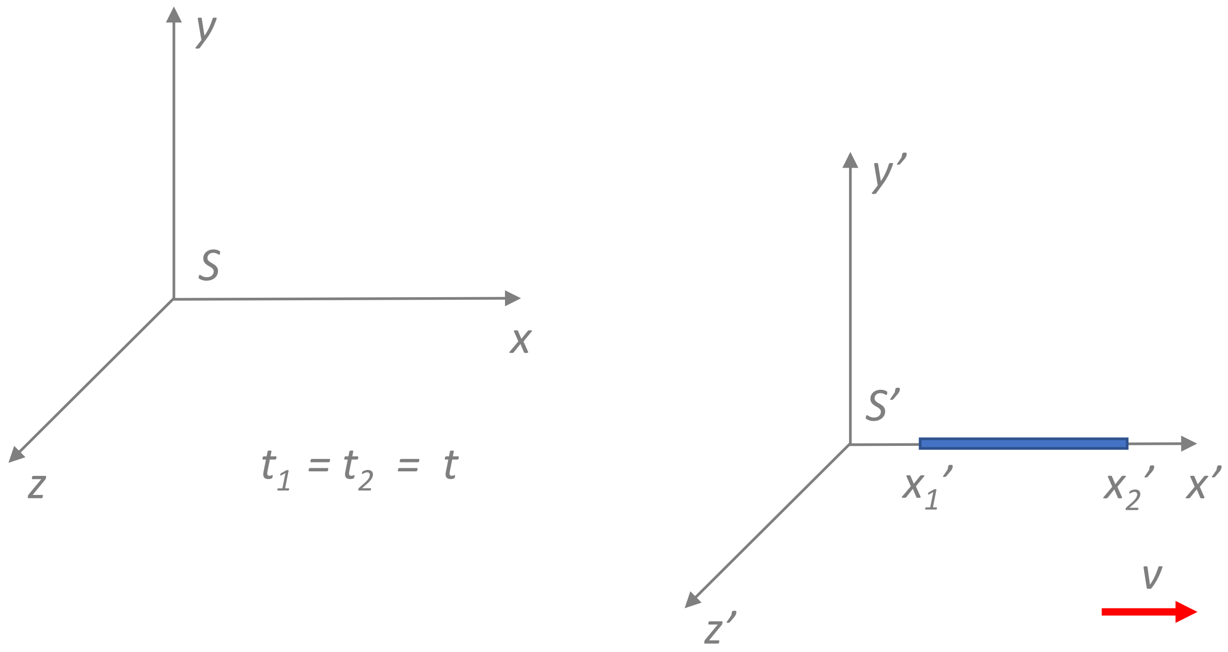
$$z = z'$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

$$k = \frac{1}{\sqrt{1-\beta^2}}$$

$$\beta = \frac{v}{c}$$

Lorentz-FitzGerald Contraction



$$L_0 = L' = x_2' - x_1'$$

$$= \gamma(x_2 - vt) - \gamma(x_2 - vt)$$

$$= \gamma(x_2 - x_1)$$

$$= \gamma L$$

$$\Downarrow$$

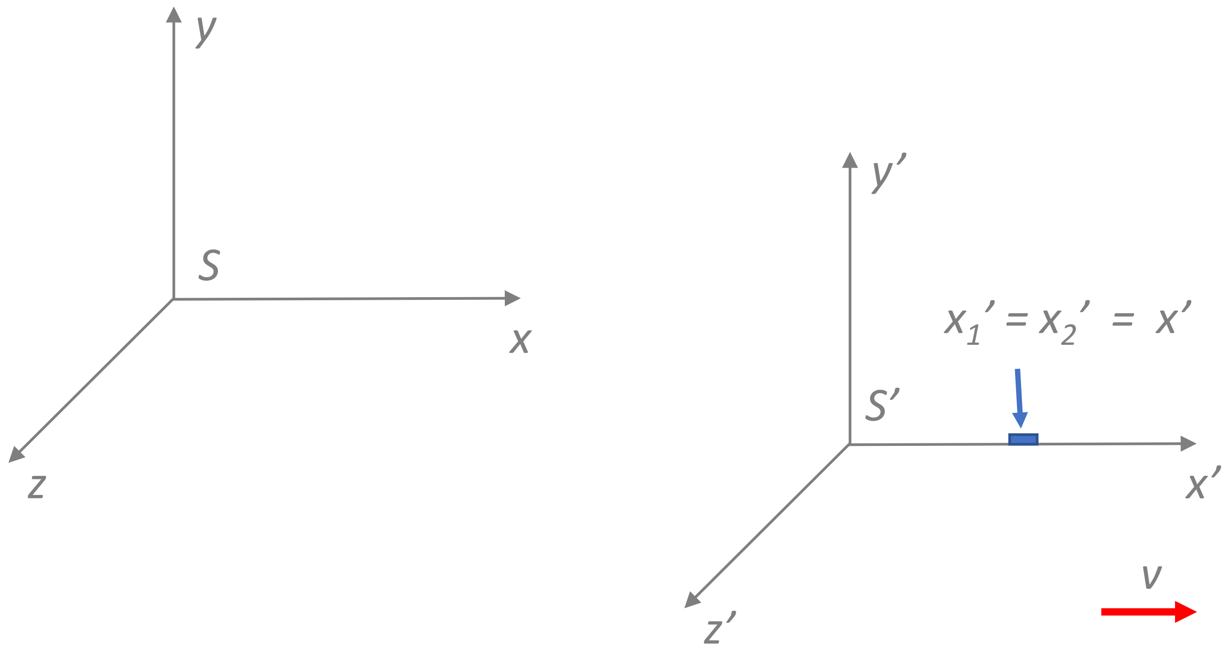
$$L = \frac{1}{\gamma} L_0$$

$$\boxed{L = \frac{1}{\gamma} L_0}$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\beta = \frac{v}{c}$$

Time Dilation



$$t = t_2 - t_1$$

$$= \gamma(t_2' + \frac{vx'}{c^2}) - \gamma(t_1' + \frac{vx'}{c^2})$$

$$= \gamma(t_2' - t_1')$$

$$= \gamma t'$$

$$= \gamma t_0$$

$$\Downarrow$$

$$t = \gamma t_0 = \gamma \tau$$

$$t = \gamma t_0 = \gamma \tau$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\beta = \frac{v}{c}$$

Velocity Addition

$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma(t - \frac{vx}{c^2})$$

$$V_x = \frac{dx}{dt} \qquad V_x' = \frac{dx'}{dt'}$$
$$V_y = \frac{dy}{dt} \qquad V_y' = \frac{dy'}{dt'}$$
$$V_z = \frac{dz}{dt} \qquad V_z' = \frac{dz'}{dt'}$$

$$dx' = \gamma(dx - vdt)$$
$$dy' = dy$$
$$dz' = dz$$
$$dt' = \gamma(dt - \frac{v}{c^2}dx)$$

$$V_x' = \frac{dx'}{dt'} = \frac{dx - vdt}{dt - \frac{v}{c^2}dx} = \frac{\frac{dx}{dt} - v}{1 - \frac{v}{c^2}\frac{dx}{dt}} = \frac{V_x - v}{1 - \frac{v}{c^2}V_x}$$

$$V_x = \frac{V_x' + v}{1 + \frac{v}{c^2}V_x'}$$

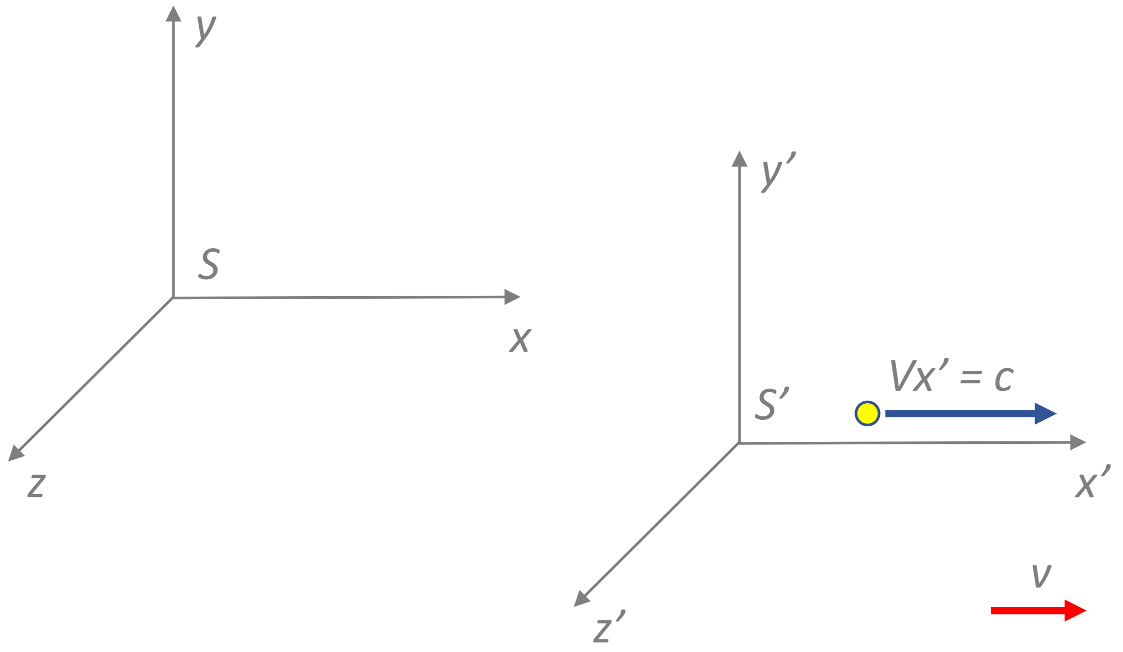
$$V_y' = \frac{dy'}{dt'} = \frac{1}{\gamma}\frac{dy}{dt - \frac{v}{c^2}dx} = \frac{1}{\gamma}\frac{\frac{dy}{dt}}{1 - \frac{v}{c^2}\frac{dx}{dt}} = \frac{1}{\gamma}\frac{V_y - v}{1 - \frac{v}{c^2}V_x}$$

$$V_y = \frac{1}{\gamma}\frac{V_y' + v}{1 + \frac{v}{c^2}V_x'}$$

$$V_z' = \frac{dz'}{dt'} = \frac{1}{\gamma}\frac{dz}{dt - \frac{v}{c^2}dx} = \frac{1}{\gamma}\frac{\frac{dz}{dt}}{1 - \frac{v}{c^2}\frac{dx}{dt}} = \frac{1}{\gamma}\frac{V_z - v}{1 - \frac{v}{c^2}V_x}$$

$$V_z = \frac{1}{\gamma}\frac{V_z' + v}{1 + \frac{v}{c^2}V_x'}$$

Velocity Addition



$$V_x' = c$$

$$\Downarrow$$

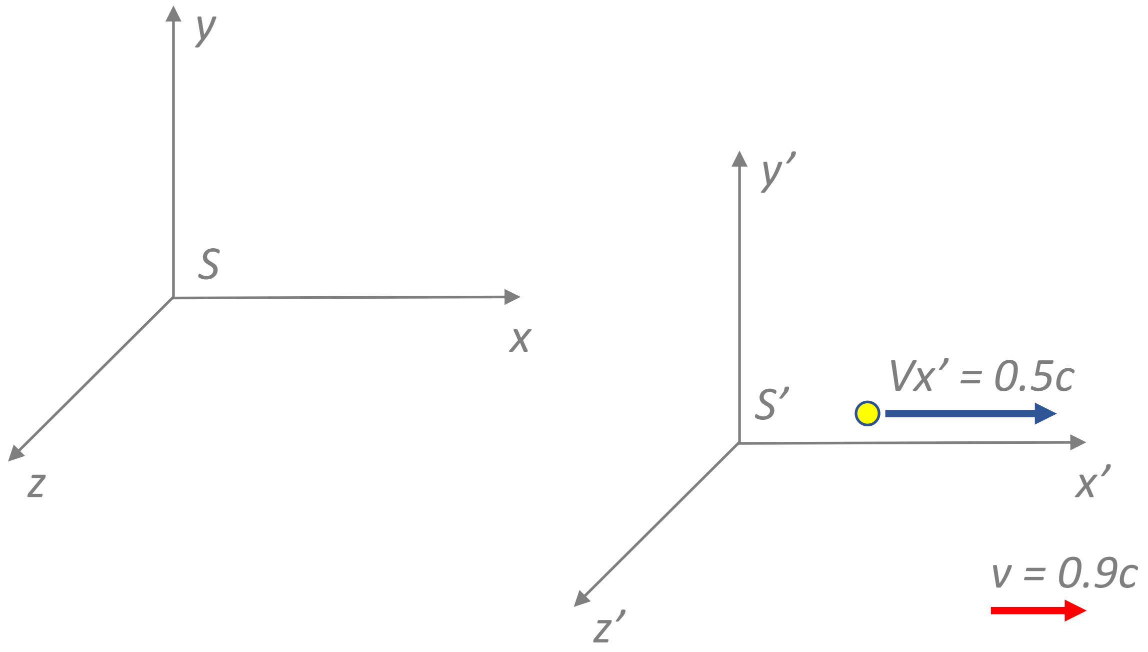
$$V_x = \frac{V_x' + v}{1 + \frac{v}{c^2} V_x'}$$

$$= \frac{c + v}{1 + \frac{v}{c^2} c}$$

$$= \frac{c + v}{1 + \frac{v}{c}}$$

$$= \frac{c(c + v)}{c + v} = c$$

Velocity Addition



$$v = 0.9c$$

$$V_x' = 0.5c$$

$$\Downarrow$$

$$V_x = \frac{V_x' + v}{1 + \frac{v}{c^2} V_x'}$$

$$= \frac{0.5c + 0.9}{1 + \frac{0.9c}{c^2} \cdot 0.5c} = 0.9665c$$

Relativity of Mass, Momentum and Force

1. Newtons 2.law (common form) $\vec{F} = m\vec{a}$
2. Newtons 2.law (general form) $\vec{F} = \frac{d}{dt}(m\vec{v})$
3. Newtons 2.law expressed by momentum $\vec{F} = \frac{d}{dt}(\vec{p}) \qquad \vec{p} = m\vec{v}$
4. Momentum $\vec{p} = m\vec{v} \qquad \vec{v} = \frac{d\vec{x}}{dt}$
5. Momentum (non-invariant) $\vec{p} = m\frac{d\vec{x}}{dt}$
6. Momentum by proper time (invariant) $\vec{p} = m_0\frac{d\vec{x}}{d\tau}$
7. Momentum relativistic (invariant) $\vec{p} = m_0\frac{d\vec{x}}{d\tau} = m_0\frac{d\vec{x}}{dt}\frac{dt}{d\tau} = m_0\frac{d\vec{x}}{dt}\frac{d}{d\tau}(\gamma\tau) = m_0\vec{v}\gamma = \gamma m_0\vec{v} = m\vec{v}$

$$m = \gamma m_0$$
$$\vec{p} = \gamma m_0\vec{v} = m\vec{v}$$

Relativity of Mass, Momentum and Force - Force

$$\vec{F} = \frac{d}{dt}(\vec{p}) = \frac{d}{dt}(\gamma m_0 \vec{v}) = \frac{d\gamma}{dt} m_0 \gamma + \gamma m_0 \frac{d\vec{v}}{dt} = m_0 \vec{v} \frac{d}{dt}\left[(1-\frac{v^2}{c^2})^{-\frac{1}{2}}\right] + \gamma m_0 \vec{a} = m_0 \vec{v}\left[\frac{v}{c^2}(1-\frac{v^2}{c^2})^{-\frac{3}{2}}(-\frac{2v}{c^2})\frac{d\vec{v}}{dt}\right] + \gamma m_0 \vec{a}$$

$$= \gamma m_0 \vec{a}\left[\frac{\frac{v^2}{c^2}}{1-\frac{v^2}{c^2}} + 1\right] = \gamma m_0 \vec{a}\left[\frac{1}{1-\frac{v^2}{c^2}}\right]$$

$$= \gamma^3 m_0 \vec{a}$$

$$\vec{F} = \gamma^3 m_0 \vec{a}$$

Relativity of Energy

$$E_K = \int_0^v F dx$$

$$= \int_0^v \gamma^3 m_0 a dx = \int_0^v \gamma^3 m_0 \frac{dv}{dt} dx = \int_0^v \gamma^3 m_0 \frac{dx}{dt} dv = \int_0^v \gamma^3 m_0 v dv$$

$$= \int_0^v (1 - \frac{v^2}{c^2})^{-\frac{3}{2}} m_0 v dv = -\frac{m_0 c^2}{2} \int_1^{1-\frac{v^2}{c^2}} u^{-\frac{3}{2}} du$$

$$= \gamma m_0 c^2 - m_0 c^2 = mc^2 - m_0 c^2 = E_{Total} - E_{\mathrm{Re}st}$$

$$E_{Total} = mc^2$$

$$E_{\mathrm{Re}st} = m_0 c^2$$

$$E_{Total} = E_K + E_{\mathrm{Re}st}$$

Relativity of Energy and Moment

$$p = mv = \gamma m_0 v$$

$$p^2 = (\gamma m_0 v) = \gamma^2 m_0{}^2 v^2 = \frac{1}{1-\frac{v^2}{c^2}} m_0{}^2 v^2$$

$$\Downarrow$$

$$v^2 = \frac{p^2 c^2}{p^2 + m_0{}^2 c^2}$$

$$\Downarrow$$

$$\gamma^2 = \frac{1}{1-\frac{v^2}{c^2}} = \frac{1}{1-\frac{\frac{p^2c^2}{p^2+m_0{}^2c^2}}{c^2}} = \frac{1}{1-\frac{p^2}{p^2+m_0{}^2c^2}} = 1+\frac{p^2}{m_0{}^2c^2}$$

$$E^2 = (mc^2)^2 = (\gamma m_0 c^2)^2 = \gamma^2 m_0{}^2 c^4 = \left[1+\frac{p^2}{m_0{}^2c^2}\right] m_0{}^2 c^4 = m_0{}^2 c^4 + p^2 c^2$$

$$\boxed{E^2 = p^2c^2 + m_0{}^2 c^4}$$

Relativity of Energy and Moment

Special Cases

1. **Massless particle:** $E^2 = p^2c^2 + m_0{}^2c^4 \qquad m_0 = 0 \;\Rightarrow\; E = pc$

1. **Kinetic energy by low speed:**

$$
\begin{aligned}
E_K &= mc^2 - m_0c^2 \\
&= \gamma m_0c^2 - m_0c^2 \\
&= m_0c^2\left[(1-\frac{v^2}{c^2})^{-\frac{1}{2}} - 1\right] \\
&\approx m_0c^2\left[1-(-\frac{1}{2}\frac{v^2}{c^2}) - 1\right] \\
&= \frac{1}{2}m_0v^2
\end{aligned}
$$

Lorentz Transformation

SpaceTime - WorldLine

Lorentz Transformation:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

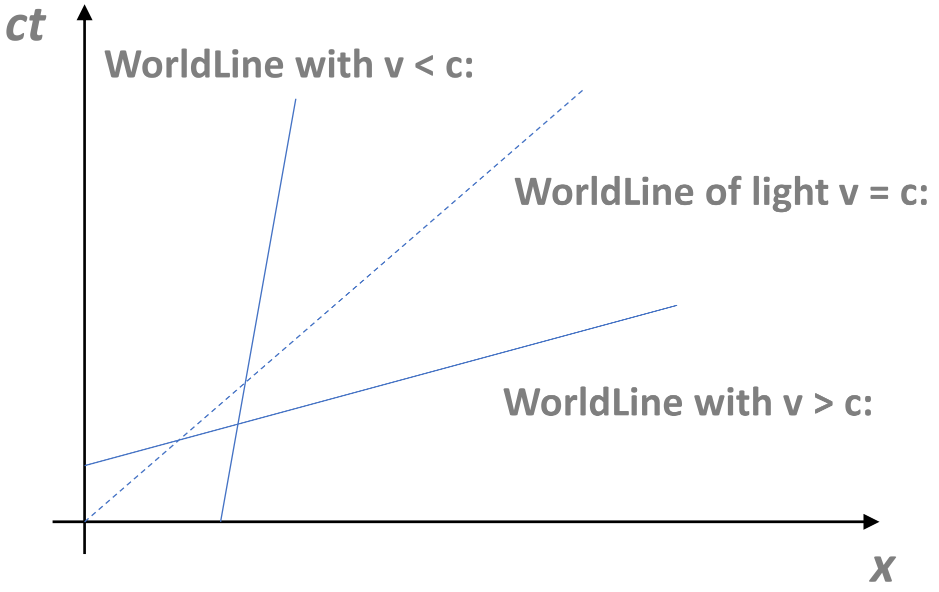
$$x' = \gamma(x - vt) \qquad \beta = \frac{v}{c}$$

$$y' = y$$

$$z' = z$$

Distance of light:

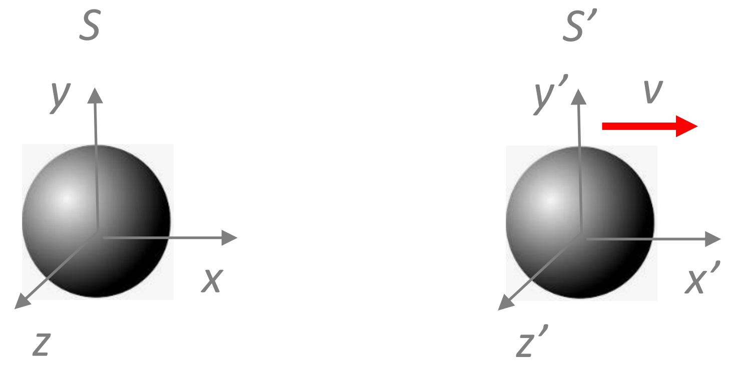
$$x = ct$$



Sphere of light:

$$x^2 + y^2 + z^2 = (ct)^2$$

$$c^2t^2 - x^2 - y^2 - z^2 = 0$$



Lorentz Transformation

Invariance

Lorentz Transformation:

$$t' = \gamma(t - \frac{vx}{c^2})$$
$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$
$$\beta = \frac{v}{c}$$

$$ct' = \gamma(ct - \frac{vx}{c})$$
$$x' = \gamma(x - vt)$$
$$y' = y$$
$$z' = z$$

Distance Invariance:

$$\Delta s'^2 = (ct')^2 - x'^2 - y'^2 - z'^2$$
$$= \left[\gamma(ct - \frac{vx}{c})\right]^2 - \left[\gamma(x - vt)\right]^2 - y^2 - z^2$$
$$= \gamma^2 c^2 t^2 - 2\gamma^2 vtx + \gamma^2 \frac{v^2}{c^2} x^2 - \gamma^2 x^2 + 2\gamma vtx - \gamma^2 v^2 t^2 - y^2 - z^2$$
$$= -\gamma^2 x^2 \left[1 - \frac{v^2}{c^2}\right] + \gamma^2 t^2 \left[c^2 - v^2\right] - y^2 - z^2$$
$$= -\gamma^2 x^2 \frac{1}{\gamma^2} + \gamma^2 t^2 \frac{c^2}{\gamma^2} - y^2 - z^2$$
$$= -x^2 + c^2 t^2 - y^2 - z^2$$
$$= c^2 t^2 - x^2 - y^2 - z^2$$
$$= \Delta s^2$$

Lorentz Transformation

Four-Vector - Matrix

Lorentz Transformation:

$$t' = \gamma(t - \frac{vx}{c^2})$$

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\beta = \frac{v}{c}$$

Four-Vector:

$$x^\mu = \begin{bmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{bmatrix} = \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

Matrix:

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

Lorentz Transformation

Matrix

Lorentz Matrix:

$$L^{\mu}{}_{\nu} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Row: Upper index μ
Col : Lower index ν

Four-Vector:

$$x^{\mu} = \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

Matrix Form:

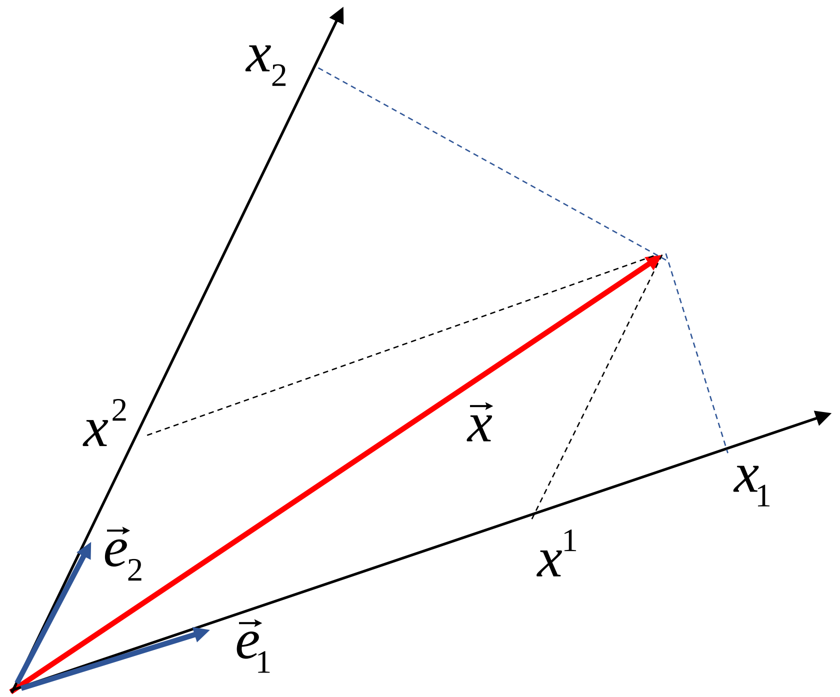
$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

$$\vec{x}\,' = L\vec{x}$$

$$x^{\mu'} = \sum_{\nu=0}^{\nu=3} L^{\mu}{}_{\nu} x^{\nu} \equiv L^{\mu}{}_{\nu} x^{\nu}$$

$$\boxed{x^{\mu'} = L^{\mu}{}_{\nu} x^{\nu}}$$

Contravariant / Covariant Vector



$$\vec{x} = x^1\vec{e}_1 + x^2\vec{e}_2 = \left[x^1, x^2\right]$$

$$x_1 = \vec{x}\cdot\vec{e}_1$$
$$x_2 = \vec{x}\cdot\vec{e}_2$$

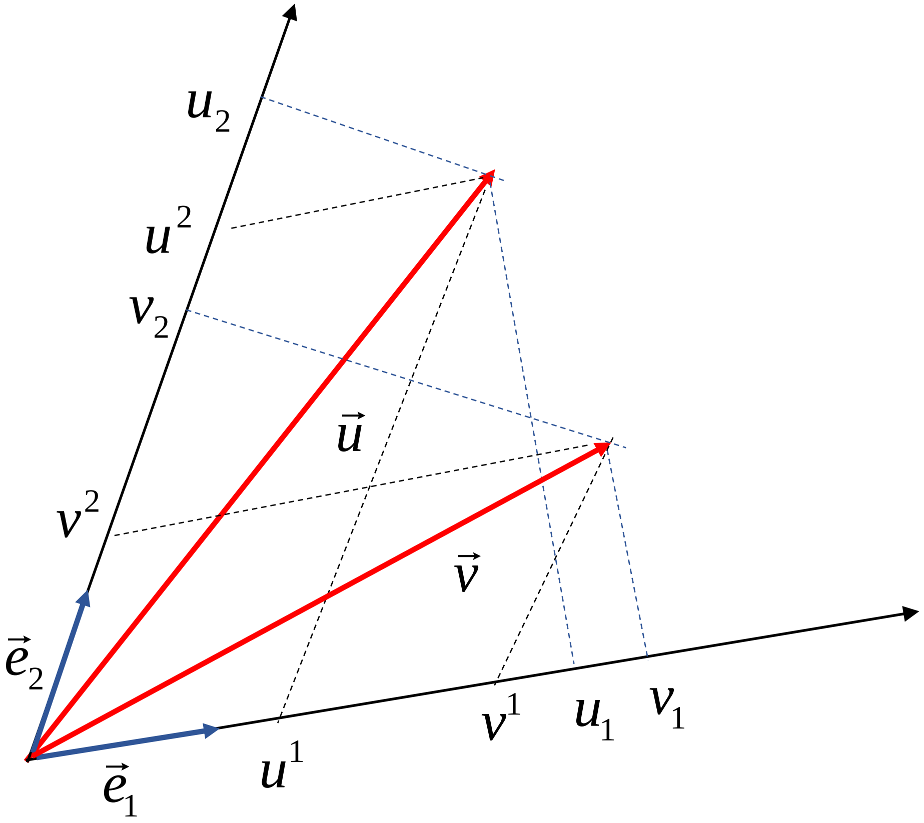
x^1, x^2 The contravariant components of the vector

x_1, x_2 The covariant components of the vector

If we increase the length of the basis vectors, then the contravariant components decrease.

If we increase the length of the basis vectors, then the covariant components also increase.

Tensor



Tensor of rank 2

$$\begin{bmatrix} T^{11} & T^{12} \\ T^{21} & T^{22} \end{bmatrix} = \begin{bmatrix} u^1v^1 & u^1v^2 \\ u^2v^1 & u^2v^2 \end{bmatrix}$$

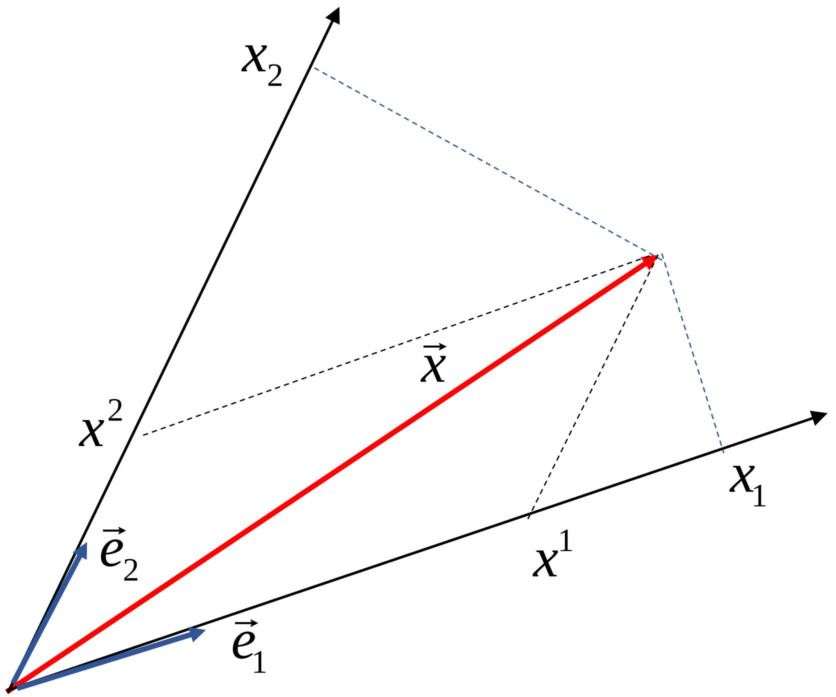
$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} u_1v_1 & u_1v_2 \\ u_2v_1 & u_2v_2 \end{bmatrix}$$

$$\begin{bmatrix} T_1^{\ 1} & T_1^{\ 2} \\ T_2^{\ 1} & T_2^{\ 2} \end{bmatrix} = \begin{bmatrix} u_1v^1 & u_1v^2 \\ u_2v^1 & u_2v^2 \end{bmatrix}$$

$$\begin{bmatrix} T^1_{\ 2} & T^1_{\ 2} \\ T^2_{\ 1} & T^2_{\ 2} \end{bmatrix} = \begin{bmatrix} u^1v_2 & u^1v_2 \\ u^2v_1 & u^2v_2 \end{bmatrix}$$

Tensor of rank 0 is a scalar
Tensor of rank 1 associate a number for each komponent (vector)
Tensor of rank 2 associate a number of each combination of 2 basis vectors
…
Tensor of rank *r* associate a number of each combination of *r* basis vectors

Contravariant / Covariant Vector



$$\vec{x} = x^1\vec{e}_1 + x^2\vec{e}_2 = \left[x^1, x^2\right] \qquad x^\mu = \sum_{\mu=1}^{2} e_\mu x^\mu = e_\mu x^\mu$$

$$x_1 = \vec{v}\cdot\vec{e}_1$$
$$x_2 = \vec{v}\cdot\vec{e}_2$$

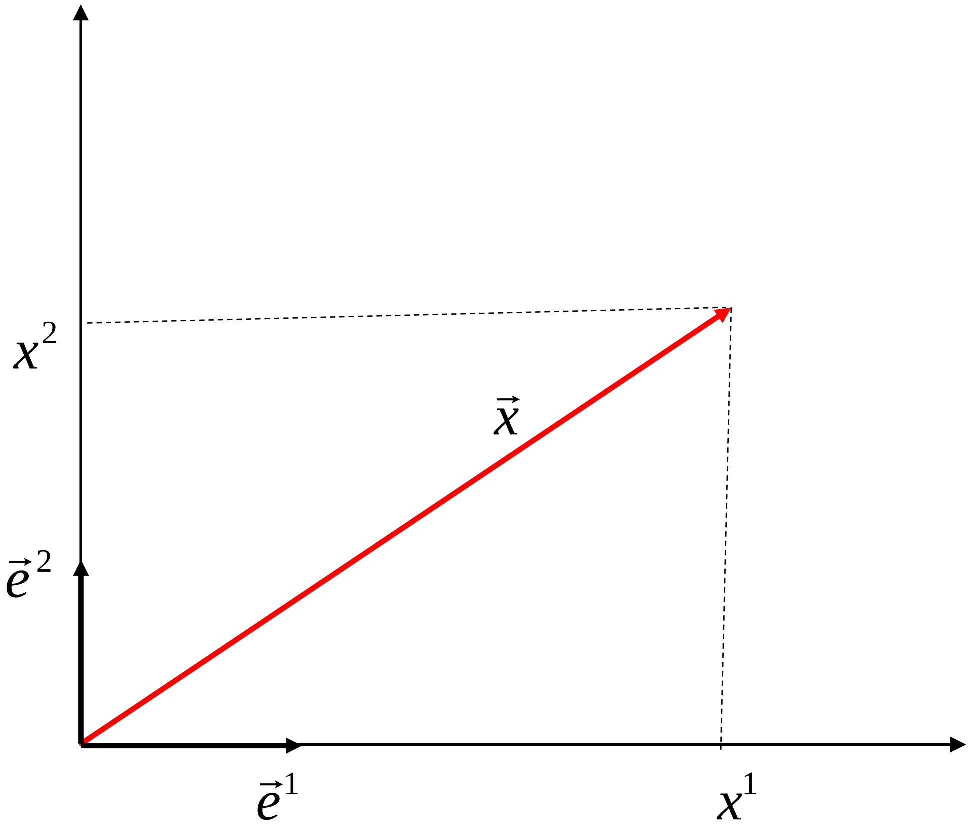
x^1, x^2 The contravariant components of the vector

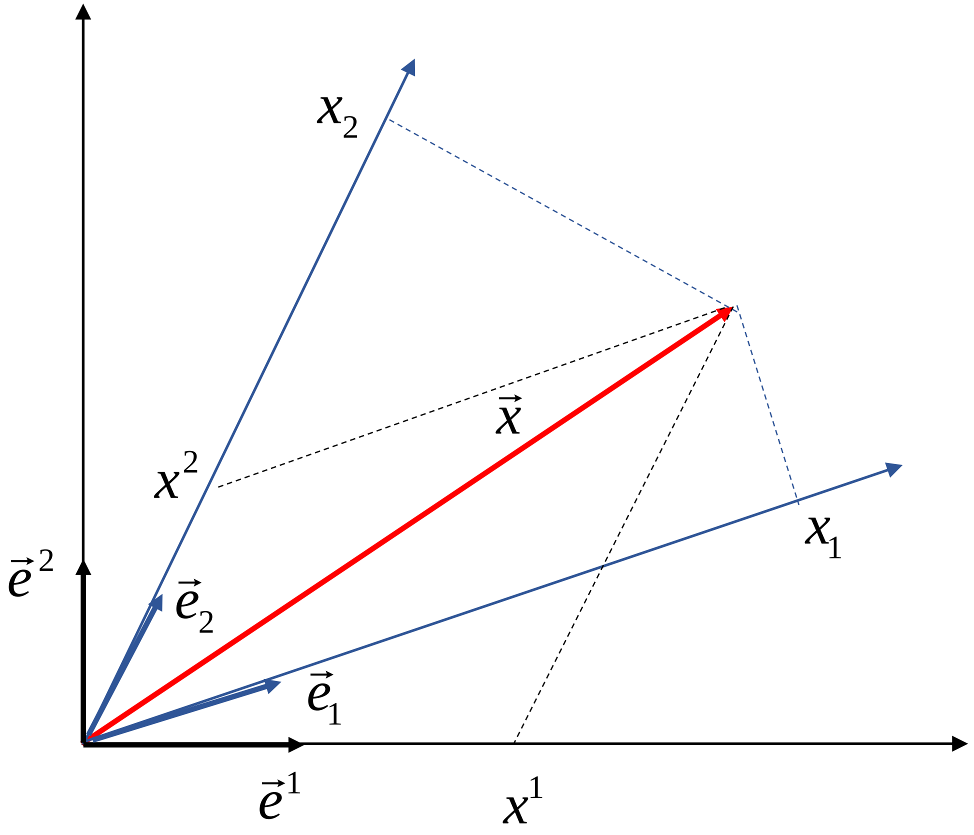
x_1, x_2 The covariant components of the vector

If we increase the length of the basis vectors, then the contravariant components decrease.

If we increase the length of the basis vectors, then the covariant components also increase.

Contravariant / Covariant Vector





$$\vec{x} = x^1\vec{e}^{\,1} + x^2\vec{e}^{\,2} = \left[x^1, x^2\right]$$

$$\vec{x} = x_1\vec{e}_1 + x_2\vec{e}_2 = \left[x_1, x_2\right]$$

Contravariant Relativity

Lorentz Transformation:

$$t' = \gamma(t - \frac{vx}{c^2})$$

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\beta = \frac{v}{c}$$

Matrix Form:

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

Four-Vector:

$$x^{\mu} = \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

Contravariant Form:

$$x^{\mu'} = \sum_{\nu=0}^{\nu=3} \Lambda_{\nu}^{\mu'} x^{\nu}$$

Compact Covariant Form:

$$x^{\mu'} = \Lambda_{\nu}^{\mu'} x^{\nu}$$

Interval:

$$\Delta x^2 = \eta_{\mu\nu} \Delta x^{\mu} \Delta x^{\nu}$$

Contravariant Relativity

Contravariant: $x^{\mu} = \left[x^0, x^1, x^2, x^3 \right]$

Covariant: $x_{\mu} = \left[x^0, -x^1, -x^2, -x^3 \right]$

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = (g^{\mu\nu}) = (g_{\mu\nu})$$

$$\vec{x} \;=\; \left[x^0, x^1, x^2, x^3 \right]$$

$$x^{\mu} \;:\; \left[x^0, x^1, x^2, x^3 \right]$$

$$x_{\mu} \;:\; \left[x^0, -x^1, -x^2, -x^3 \right]$$

$$x^{\mu'} = \sum_{\nu=0}^{\nu=3} \Lambda^{\mu'}_{\nu} x^{\nu} \equiv \Lambda^{\mu'}_{\nu} x^{\nu}$$

$$u^{\mu} v_{\mu} = u^{\mu} g_{\mu\nu} v^{\nu} = u^0 v^0 - u^1 v^1 - u^2 v^2 - u^3 v^3$$

Compact Covariant Form: $x^{\mu'} = \Lambda^{\mu'}_{\nu} x^{\nu}$

$$x_{\mu} = \sum_{\nu=0}^{\nu=3} g_{\mu\nu} x^{\nu} \equiv g_{\mu\nu} x^{\nu}$$

Interval: $\Delta x^2 = \eta_{\mu\nu} \Delta x^{\mu} \Delta x^{\nu}$

Contravariant Relativity

$$T^{\mu}{}_{\nu} = T^{\mu\lambda} g_{\lambda\nu} = \begin{bmatrix} T^{00} & -T^{01} & -T^{02} & -T^{03} \\ T^{10} & -T^{11} & -T^{12} & -T^{13} \\ T^{20} & -T^{21} & -T^{22} & -T^{23} \\ T^{30} & -T^{31} & -T^{32} & -T^{33} \end{bmatrix} = g_{\nu\lambda} T^{\mu\lambda}$$

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

$$T_{\mu}{}^{\nu} = g_{\mu\lambda} T^{\lambda\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ -T^{10} & -T^{11} & -T^{12} & -T^{13} \\ -T^{20} & -T^{21} & -T^{22} & -T^{23} \\ -T^{30} & -T^{31} & -T^{32} & -T^{33} \end{bmatrix}$$

μ row
ν column

$$T_{\mu\nu} = g_{\mu\kappa} T^{\kappa\lambda} g_{\lambda\mu} = \begin{bmatrix} T^{00} & -T^{01} & -T^{02} & -T^{03} \\ -T^{10} & T^{11} & T^{12} & T^{13} \\ -T^{20} & T^{21} & T^{22} & T^{23} \\ -T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$