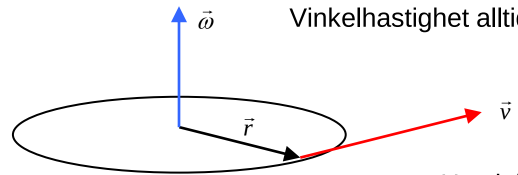
Kap 09 Rotasjon

Vinkelhastighet og vinkelakselerasjon som vektor



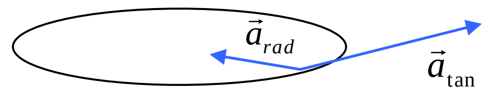
Vinkelhastighet alltid normal på rotasjonsplanet

Hastighet alltid tangentiell til banen

Hastighet $\quad \vec{v} = \vec{\omega} \times \vec{r}$

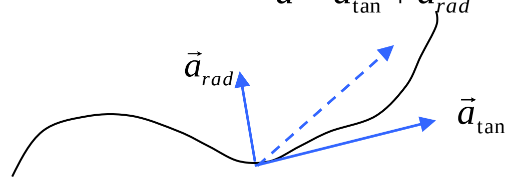
Akselerasjon

$$\begin{aligned}
\vec{a} &= \dot{\vec{v}} \\
&= \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times \dot{\vec{r}} \\
&= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \dot{\vec{r}} \\
&= \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v} \\
&= \underbrace{\vec{\alpha} \times \vec{r}}_{\substack{\text{tangensiell} \\ \text{akselerasjon}}} + \underbrace{\vec{\omega} \times \vec{\omega} \times \vec{r}}_{\substack{\text{radielll} \\ \text{akselerasjon}}} \\
&= \vec{a}_{\tan} + \vec{a}_{rad}
\end{aligned}$$

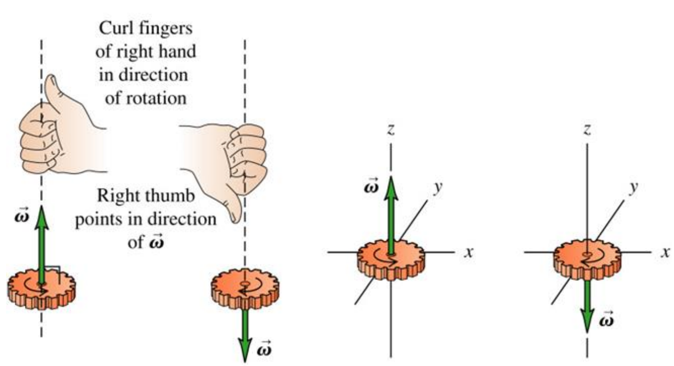


Tangentiell-akselerasjon alltid tangentiell til banen.
Radiell-akselerasjon alltid rettet inn mot sentrum.

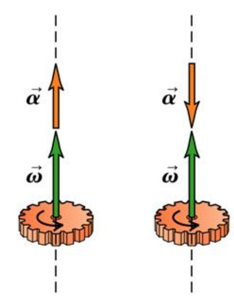
$$\vec{a} = \vec{a}_{\tan} + \vec{a}_{rad}$$



$$a = \sqrt{a_{\tan}^{\;2} + a_{rad}^{\;2}}$$



(a) (b) $\omega_z > 0$ (c) $\omega_z < 0$



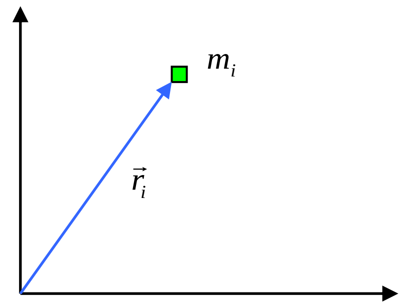
Speeding up Slowing up

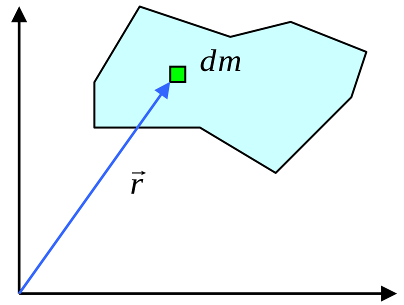
Kap 09 Rotasjon

Treghetsmoment

$$I = \sum_i m_i r_i^2$$

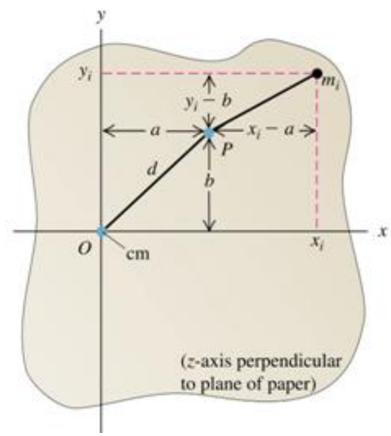
$$I = \int r^2 dm$$





Kinetisk rotasjonsenergi

$$K = \frac{1}{2} I \omega^2$$



Parallellakse-teorem

$$I_P = I_{cm} + Md^2$$

Benevning

$I \leftrightarrow kgm^2$

$K \leftrightarrow J$

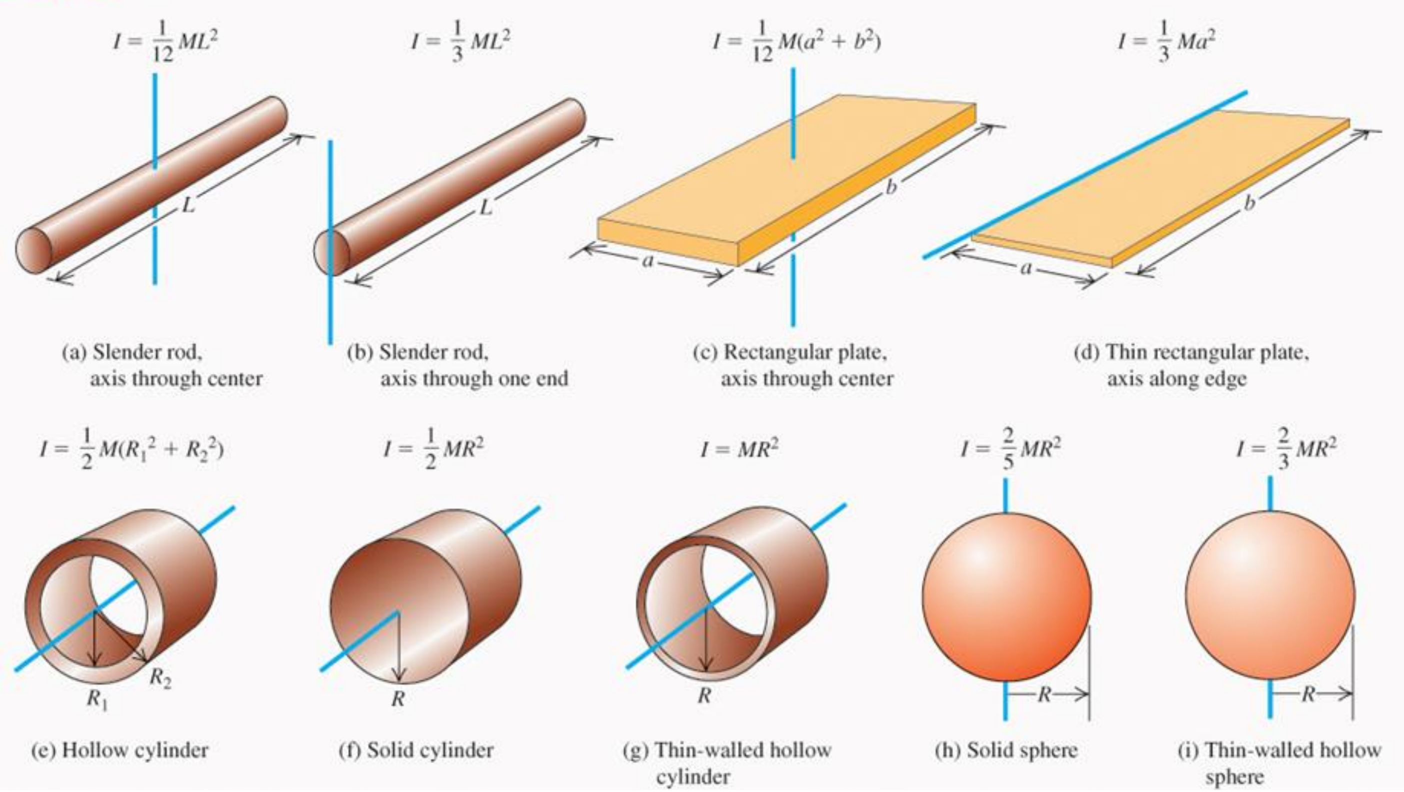
Normalakse-teorem

$$I_O = I_x + I_y$$

Treghets-moment for noen spesielle legemer med akse gjennom sentrum

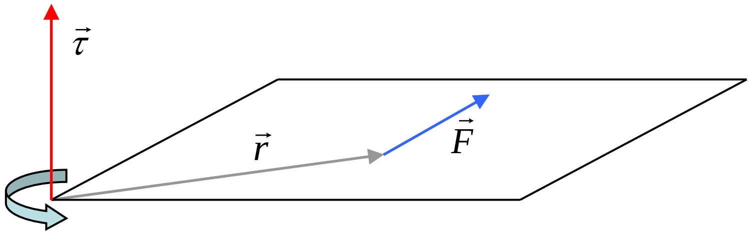
Stav med masse M og lengde L	$I = \frac{1}{12} ML^2$
Rektangulær plate med masse M og sider a og b	$I = \frac{1}{12} M(a^2 + b^2)$
Hul sylinder med masse M, indre radius R_1, ytre radius R_2	$I = \frac{1}{2} M(R_1{}^2 + R_2{}^2)$
Massiv sylinder med masse M og radius R	$I = \frac{1}{2} MR^2$
Massiv kule med masse M og radius R	$I = \frac{2}{5} MR^2$
Tynnvegget kule med masse M og radius R	$I = \frac{2}{3} MR^2$

Table 9.2 Moments of Inertia of Various Bodies



Kap 10   Rotasjonsdynamikk

Kraftmoment / Angulært moment



Kraftmoment $\vec{\tau} = \vec{r} \times \vec{F}$

$\tau_{cm} = I_{cm}\alpha$

$\tau_o = I_o\alpha$ når $\begin{cases} 1. & \vec{a}_o = \vec{0} \\ 2. & \vec{a}_o \parallel \vec{r}_{cm} \\ 3. & \vec{r}_{cm} = \vec{0} \end{cases}$

Kinetisk energi $K = K_{trans} + K_{rot} = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$

$K = \frac{1}{2}I_o\omega^2$ når $v_o = \vec{0}$

Arbeid / Effekt $W = \int_{\theta_1}^{\theta_2} \tau d\theta = \int_{\omega_1}^{\omega_2} I\omega d\omega = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2$

$P = \frac{dW}{dt} = \tau\frac{d\theta}{dt} = \tau\omega$

Angulært moment $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$

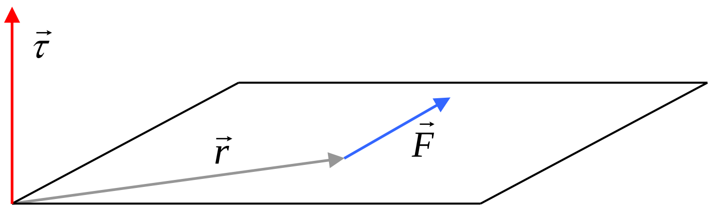
$\vec{L} = I\vec{\omega}$

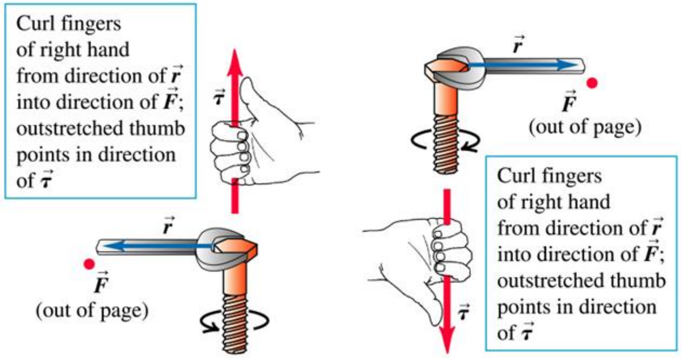
$\dot{\vec{L}}_o = \vec{\tau}_o$ når $\begin{cases} 1. & \vec{v}_o = \vec{0} \\ 2. & \vec{v}_{cm} = \vec{0} \\ 3. & \vec{v}_o \parallel \vec{v}_{cm} \end{cases}$

Gyroskop $\Omega = \frac{d\varphi}{dt} = \frac{\frac{dL}{L}}{dt} = \frac{\frac{dL}{dt}}{L} = \frac{\tau}{L} = \frac{wR}{I\omega}$

Kraftmoment

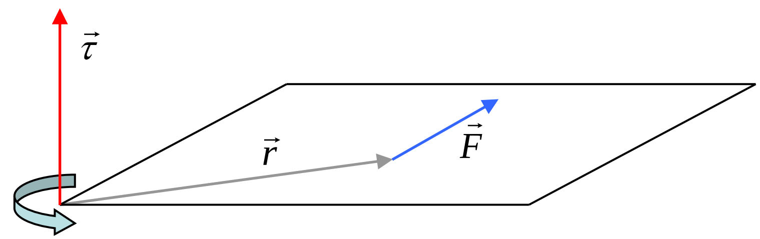
$$\vec{\tau} = \vec{r} \times \vec{F}$$





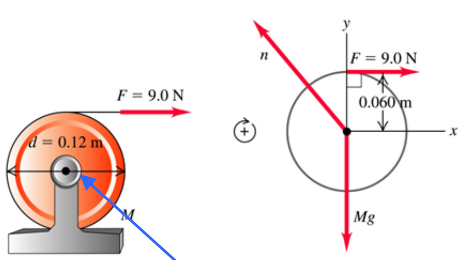
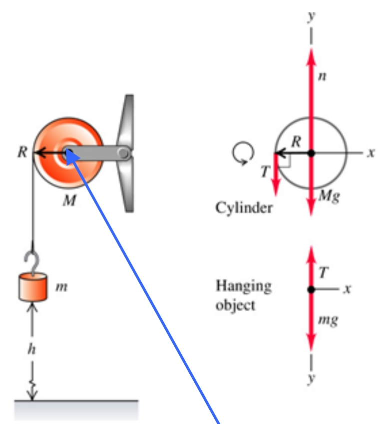
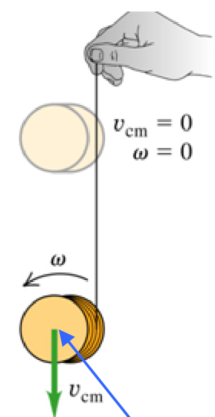
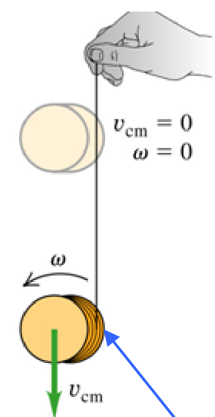
Kap 10 Rotasjonsdynamikk

Sammenheng mellom kraftmoment og vinkel-akselerasjon

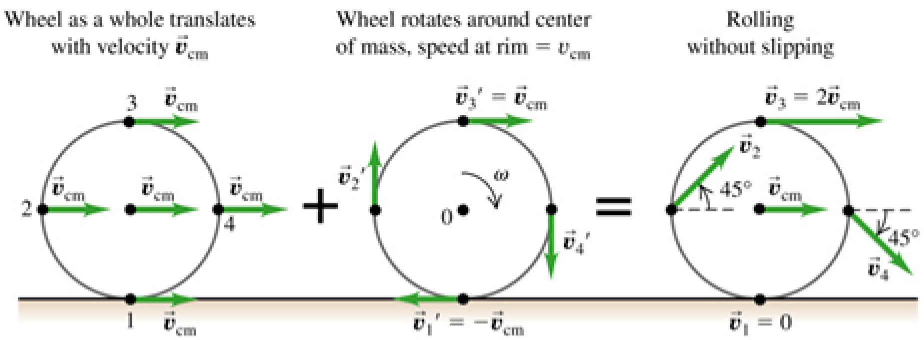


$$\tau_{\text{cm}} = I_{cm}\alpha$$

$$\tau_o = I_o\alpha \qquad \text{når} \quad \begin{cases} 1. \quad \vec{a}_o = \vec{0} \\ 2. \quad \vec{a}_o \parallel \vec{r}_{cm} \\ 3. \quad \vec{r}_{cm} = \vec{0} \end{cases}$$

1		2	3
			

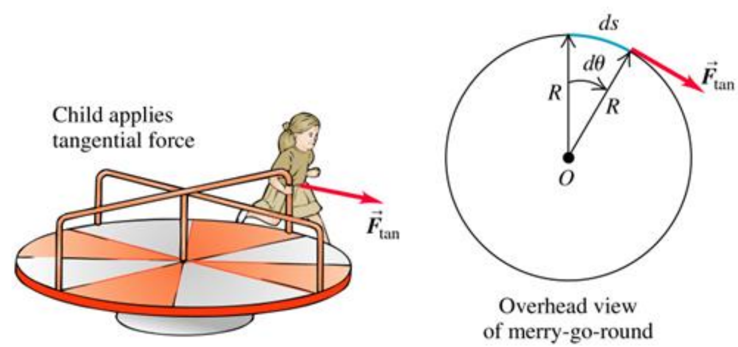
Kap 10 Rotasjonsdynamikk



Kinetisk energi

$$K = K_{trans} + K_{rot} = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I_{cm} \omega^2$$

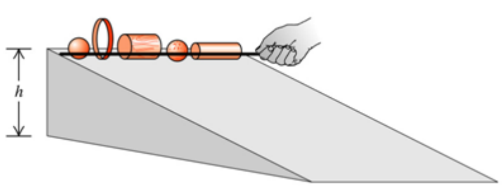
$$K = \frac{1}{2} I_o \omega^2 \qquad \text{når} \quad v_o = \vec{0}$$



Arbeid / Effekt

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta = \int_{\omega_1}^{\omega_2} I \omega d\omega = \frac{1}{2} I \omega_2^2 - \frac{1}{2} I \omega_1^2$$

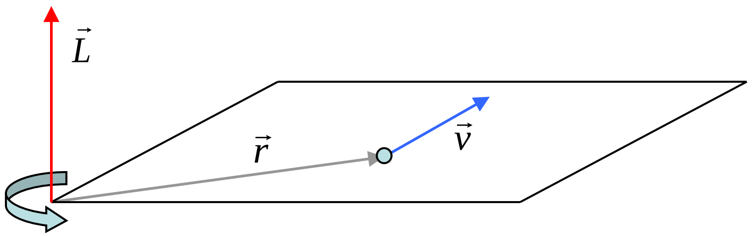
$$P = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau\omega$$



$$0 + M \cdot g \cdot h = \frac{1}{2} \cdot M \cdot v_{cm}^2 + \frac{1}{2} \cdot \left(c \cdot M \cdot R^2\right) \cdot \left(\frac{v_{cm}}{R}\right)^2$$

$$M \cdot g \cdot h = \frac{M \cdot v_{cm}^2 \cdot (c+1)}{2} \qquad v_{cm} = \sqrt{\frac{2 \cdot g \cdot h}{c+1}}$$

Kap 10 Rotasjonsdynamikk



Angulært moment $\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v}$

$$\vec{L} = I\vec{\omega}$$

$$\dot{\vec{L}}_o = \vec{\tau}_o \qquad \text{når} \quad \begin{cases} 1. \quad \vec{v}_o = \vec{0} \\ 2. \quad \vec{v}_{cm} = \vec{0} \\ 3. \quad \vec{v}_o \parallel \vec{v}_{cm} \end{cases}$$