UNIVERSITETET I AGDER
Grimstad

E K S A M E N S O P P G A V E :

FAG: MA-209 Matematikk 3 LÆRER: Per Henrik Hogstad

Klasse(r):	**Dato: 08.08.12**	**Eksamenstid, fra-til: 09.00 – 14.00**	
Eksamensoppgaven består av følgende	**Antall sider: 5 (inkl. forside + vedlegg)**	**Antall oppgaver: 3**	**Antall vedlegg: 1**
Tillatte hjelpemidler er:	**Kalkulator** **Hogstad: Formler MA-209** **Haugan: Formler og tabeller** **Gyldendals formelsamling** **(Ikke tillatt å skrive i formelsamlingene)**		
KANDIDATEN MÅ SELV KONTROLLERE AT OPPGAVESETTET ER FULLSTENDIG			

MA-209 Utsatt Eksamen 2012

Oppg nr		Poeng
1	a	3
	b	3
2	a	3
	b	3
	c	3
3	a	3
	b	3
	c	3
	d	3
	e	3
	f	3
Sum		33

Poengene viser vekt-fordelingen for de enkelte del-spørsmålene.
Ved karaktersetting vektlegges selvfølgelig i tillegg en total-vurdering, bl.a. en vurdering av i hvilken grad kandidaten har kunnskaper innenfor de ulike områdene gitt i oppgave-settet.

LYKKE TIL !

1. Vi har et legeme T avgrenset av de tre koordinatplanene (xy-planet, yz-planet, xz-planet), og de to planene $z = 1 + y$ og $2x + y = 2$.

   a) Tegn en skisse av legemet T.

   b) Bestem vha trippelintegral volumet V av legemet T.

2. Vi har følgende lukkede kurve C i planet (se fig 2.1):

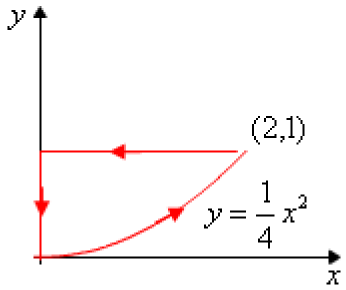


Fig 2.1

Pilene indikerer positiv omløpsretning av kurven C (retning mot klokka).
Kurven C er sammensatt av tre delkurver:

C_1: Parabelen $y = 1/4x^2$ fra punktet (0,0) til punktet (2,1).
C_2: Det horisontale linjestykket fra punktet (2,1) til punktet (0,1).
C_3: Det vertikale linjestykket fra punktet (0,1) til punktet (0,0).

Vi har videre gitt følgende vektorfelt:

$$\vec{F} = [xy, -y^2]$$

a) Bestem kurveintegralet

$$\oint_C \vec{F} \cdot d\vec{r}$$

langs den lukkede kurven C direkte uten bruk av Greens (eller Stokes) teorem.

b) Bestem kurveintegralet i a) ved hjelp av Greens teorem.

c) Bestem arealet A innenfor den lukkede kurven C ved hjelp av Greens arealteorem.

3. Vi har gitt følgende vektorfelt:

$$\vec{F} = [z - y, x, -x]$$

Videre har vi gitt følgende halvkuleflate *S* og sirkel *C* i rommet:

$$S : x^2 + y^2 + z^2 = 4 \qquad z \geq 0$$

$$C : x^2 + y^2 = 4 \qquad z = 0$$

C er orientert i positiv retning mot klokka sett ovenfra nedover langs *z*-aksen.

a) Bestem divergens og curl til det nevnte vektorfeltet.

b) Bestem kurveintegralet

$$\oint_C \vec{F} \cdot d\vec{r}$$

langs den lukkede kurven C direkte uten bruk av Stokes teorem.

c) Bestem kurveintegralet i b) ved hjelp av Stokes teorem
når det integreres over halvkuleflaten *S*.

d) Bestem kurveintegralet i b) ved hjelp av Stokes teorem
når det integreres over den sirkelskiven i *xy*-planet som er avgrenset av kurven *C*.

e) Bestem fluksen av vektorfeltet ut av halvkuleflaten *S*.

f) La *F* være et generelt vektorfelt og la *f* være et generelt skalarfelt (en generell skalarfunksjon)
med kontinuerlige partielle deriverte.
Vis at da gjelder følgende to identiteter:

$$\nabla \cdot (\nabla \times \vec{F}) = 0 \qquad (\text{div curl} = 0)$$

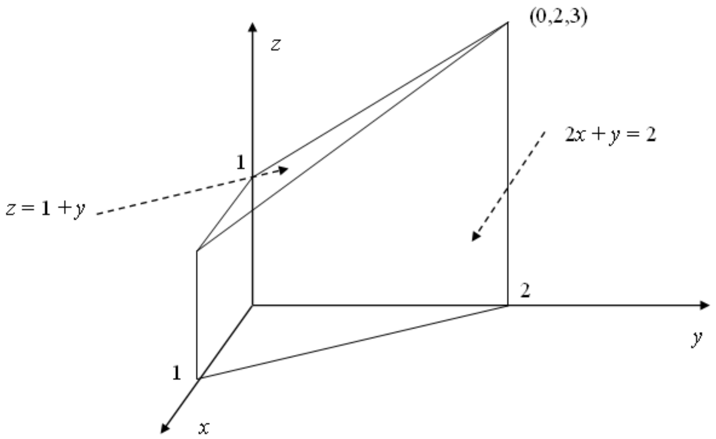
$$\nabla \times (\nabla f) = \vec{0} \qquad (\text{curl grad} = \vec{0})$$

Vedlegg:

$$\cos^2 u + \sin^2 u = 1$$
$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$
$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$
$$\cos 2u = \cos^2 u - \sin^2 u$$
$$\sin 2u = 2 \sin u \cos u$$
$$\cos^2 u = \frac{1 + \cos 2u}{2}$$
$$\sin^2 u = \frac{1 - \cos 2u}{2}$$
$$(\cos u)' = -\sin u$$
$$(\sin u)' = \cos u$$
$$e^{\ln x} = x$$
$$(e^x)' = e^x$$
$$\int u dv = uv - \int v du$$

Løsning:

1. a) Tegning av legemet T:



b) Volum V av legemet T:

$$V = \iiint_T dV = \iiint_T dxdydz = \iiint_T dzdydx$$
$$= \int_{x=0}^{x=1}\int_{y=0}^{y=2-2x}\int_{z=0}^{z=1+y} dzdydx = \int_{x=0}^{x=1}\int_{y=0}^{y=2-2x}\big[z\big]_{z=0}^{z=1+y} dydx = \int_{x=0}^{x=1}\int_{y=0}^{y=2-2x}(1+y)dydx$$
$$= \int_{x=0}^{x=1}\left[y+\frac{1}{2}y^2\right]_{y=0}^{y=2-2x} dx = \int_{x=0}^{x=1}\left[(2-2x)+\frac{1}{2}(2-2x)^2\right]dx = \int_{x=0}^{x=1}(2-2x+2-4x+2x^2)dx$$
$$= \int_{x=0}^{x=1}(2x^2-6x+4)dx = 2\int_{x=0}^{x=1}(x^2-3x+2)dx = 2\left[\frac{1}{3}x^3-\frac{3}{2}x^2+2x\right]_{x=0}^{x=1} = 2\left[\frac{1}{3}-\frac{3}{2}+2\right] = 2\cdot\frac{5}{6} = \underline{\underline{\frac{5}{3}}}$$

Eller:

$$V = \iiint_T dV = \iiint_T dxdydz = \iiint_T dzdxdy$$
$$= \int_{y=0}^{y=2}\int_{x=0}^{x=1-\frac{y}{2}}\int_{z=0}^{z=1+y} dzdxdy = \int_{=0}^{=21}\int_{x=0}^{x=1-\frac{y}{2}x}\big[z\big]_{z=0}^{z=1+y} dxdy = \int_{x=0}^{21}\int_{x=0}^{x=1-\frac{y}{2}x}(1+y)dxdy$$
$$= \int_{y=0}^{y=2}\big[(1+y)x\big]_{x=0}^{x=1-\frac{y}{2}} dy = \int_{x=0}^{x=1}(1+y)(1-\frac{y}{2})dy = \int_{x=0}^{x=1}(1+y-\frac{1}{2}y-\frac{1}{2}y^2)dy$$
$$= \int_{x=0}^{x=1}(1+\frac{y}{2}-\frac{y^2}{2})dy = \left[y+\frac{1}{4}y^2+\frac{1}{6}y^3\right]_{y=0}^{y=2} = \left[2+1-\frac{4}{3}\right] = \underline{\underline{\frac{5}{3}}}$$

2. a)

$$C_1 : \vec{r}(t) = \left[t, \frac{1}{4}t^2 \right] \qquad \text{t} \in [0,2]$$

$$d\vec{r}(t) = \left[1, \frac{1}{4} \cdot 2t \right] dt = \left[1, \frac{1}{2}t \right] dt$$

$$\vec{F}(\vec{r}(t)) = \left[t \cdot \frac{1}{4}t^2, -\left(\frac{1}{4}t^2\right)^2 \right] = \left[\frac{1}{4}t^3, -\frac{1}{16}t^4 \right]$$

$$\vec{F} \cdot d\vec{r} = \vec{F}(\vec{r}(t)) \cdot d\vec{r}(t) = \left[\frac{1}{4}t^3, -\frac{1}{16}t^4 \right] \cdot \left[1, \frac{1}{2}t \right] dt = \underline{\left(\frac{1}{4}t^3 - \frac{1}{32}t^5 \right) dt}$$

$$C_2 : \vec{r}(t) = [2-t, 1] \qquad t \in [0,2] \qquad \text{Alternativt} : \vec{r}(t) = [t, 1] \qquad t : 2 \to 0$$

$$d\vec{r}(t) = [-1, 0] dt$$

$$\vec{F}(\vec{r}(t)) = \left[(2-t) \cdot 1, -1^2 \right] = [2-t, -1]$$

$$\vec{F} \cdot d\vec{r} = \vec{F}(\vec{r}(t)) \cdot d\vec{r}(t) = [2-t, -1] \cdot [-1, 0] dt = \underline{(t-2) dt}$$

$$C_3 : \vec{r}(t) = [0, 1-t] \qquad t \in [0,1] \qquad \text{Alterntivt} : \vec{r}(t) = [0, t] \qquad t : 1 \to 0$$

$$d\vec{r}(t) = [0, -1] dt$$

$$\vec{F}(\vec{r}(t)) = \left[0 \cdot (1-t), -(1-t)^2 \right] = \left[0, -1 + 2t - t^2 \right]$$

$$\vec{F} \cdot d\vec{r} = \vec{F}(\vec{r}(t)) \cdot d\vec{r}(t) = \left[0, -1 + 2t - t^2 \right] \cdot [0, -1] dt = \underline{1 - 2t + t^2}$$

$$\oint_C \vec{F} \cdot d\vec{r} = \int_{C_1} \vec{F} \cdot d\vec{r} + \int_{C_2} \vec{F} \cdot d\vec{r} + \int_{C_3} \vec{F} \cdot d\vec{r}$$

$$= \int_{t=0}^{t=2} \left(\frac{1}{4}t^3 - \frac{1}{32}t^5 \right) dt + \int_{t=0}^{t=2} (t-2) dt + \int_{t=0}^{t=1} \left(1 - 2t + t^2 \right) dt$$

$$= \left[\frac{1}{16}t^4 - \frac{1}{192}t^6 \right]_{t=0}^{t=2} + \left[\frac{1}{2}t^2 - 2t \right]_{t=0}^{t=2} + \left[t - t^2 + \frac{1}{3}t^3 \right]_{t=0}^{t=1}$$

$$= \frac{1}{16} \cdot 2^4 - \frac{1}{192} \cdot 2^6 + \frac{1}{2} \cdot 2^2 - 2 \cdot 2 + 1 - 1^2 + \frac{1}{3} \cdot 1^3$$

$$= \frac{1}{16} \cdot 16 - \frac{1}{192} \cdot 64 + \frac{1}{2} \cdot 4 - 4 + 1 - 1 + \frac{1}{3} = 1 - \frac{1}{3} + 2 - 4 + 1 - 1 + \frac{1}{3} = \underline{\underline{-1}}$$

eller:

$$\oint_C \vec{F}\cdot d\vec{r} = \int_{C_1} \vec{F}\cdot d\vec{r} + \int_{C_2} \vec{F}\cdot d\vec{r} + \int_{C_3} \vec{F}\cdot d\vec{r}$$
$$= \int_{C_1} [F_1, F_2]\cdot[dx, dy] + \int_{C_2} [F_1, F_2]\cdot[dx, dy] + \int_{C_3} [F_1, F_2]\cdot[dx, dy]$$
$$= \int_{C_1} F_1 dx + F_2 dy + \int_{C_2} F_1 dx + F_2 dy + \int_{C_3} F_1 dx + F_2 dy$$
$$= \int_{C_1} xydx + (-y^2)dy + \int_{C_2} xydx + (-y^2)dy + \int_{C_3} xydx + (-y^2)dy$$
$$= \int_{C_1} x\frac{1}{4}x^2dx + (-y^2)dy + \int_{C_2} x\cdot 1dx + (-y^2)\cdot 0 + \int_{C_3} 0\cdot y\cdot 0 + (-y^2)dy$$
$$= \int_{C_1} \frac{1}{4}x^3dx - y^2dy + \int_{C_2} xdx - \int_{C_3} y^2dy$$
$$= \int_{x=0}^{x=2} \frac{1}{4}x^3dx - \int_{y=0}^{y=1} y^2dy + \int_{x=2}^{x=0} xdx - \int_{y=1}^{y=0} y^2dy$$
$$= \left[\frac{1}{16}x^4\right]_{x=0}^{x=2} - \left[\frac{1}{3}y^3\right]_{y=0}^{y=1} + \left[\frac{1}{2}x^2\right]_{x=2}^{x=0} - \left[\frac{1}{3}y^3\right]_{y=1}^{y=0}$$
$$= \frac{1}{16}\cdot 2^4 - \frac{1}{3}\cdot 1^3 - \frac{1}{2}\cdot 2^2 + \frac{1}{3}\cdot 1^3$$
$$= \frac{1}{16}\cdot 16 - \frac{1}{3}\cdot 1 - \frac{1}{2}\cdot 4 + \frac{1}{3}\cdot 1$$
$$= 1 - \frac{1}{3} - 2 + \frac{1}{3} = \underline{\underline{-1}}$$

b)

$$\oint_C \vec{F}\cdot d\vec{r} = \iint_R \left(\nabla\times\vec{F}\right)\cdot\vec{k}dA = \iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right)dA = \iint_R \left(\frac{\partial}{\partial x}(-y^2) - \frac{\partial}{\partial y}(xy)\right)dA$$
$$= \iint_R (0 - x)dA = -\iint_R xdA = -\int_{x00}^{x=2}\int_{y=\frac{1}{4}x^2}^{y=1} xdydx = -\int_{x00}^{x=2} [xy]_{y=\frac{1}{4}x^2}^{y=1} dx = -\int_{x00}^{x=2}\left(x - \frac{1}{4}x^3\right)dx$$
$$= -\left[\frac{1}{2}x^2 - \frac{1}{16}x^4\right]_{x=0}^{x=2} = -\left(\frac{1}{2}\cdot 2^2 - \frac{1}{16}\cdot 2^4\right) = -2 + 1 = \underline{\underline{-1}}$$

c) Areal:

$$A = \iint_R dA = \oint_C xdy = \int_{C_1} xdy + \int_{C_2} xdy + \int_{C_3} xdy$$
$$= \int_{C_1} x\frac{1}{4}\cdot 2xdx + \int_{C_2} x\cdot 0 + \int_{C_3} 0dy = \frac{1}{2}\int_{x=0}^{x=2} x^2dx = \frac{1}{2}\left[\frac{1}{3}x^3\right]_{x=0}^{x=2} = \frac{1}{2}\cdot\frac{1}{3}\cdot 2^3 = \underline{\underline{\frac{4}{3}}}$$

3. a) Vektorfelt:

$$\vec{F} = [z-y, x, -x]$$

Divergens:

$$
\begin{aligned}
div\vec{F} = \nabla \cdot \vec{F} &= \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right] \cdot [F_1, F_2, F_3] \\
&= \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} \\
&= \frac{\partial (z-y)}{\partial x} + \frac{\partial x}{\partial y} + \frac{\partial (-x)}{\partial z} \\
&= 0 + 0 + 0 \\
&= \underline{\underline{0}}
\end{aligned}
$$

Curl:

$$
\begin{aligned}
curl\vec{F} = \nabla \times \vec{F} &= \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right] \times [F_1, F_2, F_3] \\
&= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix} \\
&= \left[\frac{\partial F_3}{\partial y} - \frac{\partial F_2}{\partial z}, \frac{\partial F_1}{\partial z} - \frac{\partial F_3}{\partial x}, \frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right] \\
&= \left[\frac{\partial (-x)}{\partial y} - \frac{\partial x}{\partial z}, \frac{\partial (z-y)}{\partial z} - \frac{\partial (-x)}{\partial x}, \frac{\partial x}{\partial x} - \frac{\partial (z-y)}{\partial y}\right] \\
&= [0 - 0, 1 - (-1), 1 - (-1)] \\
&= \underline{\underline{[0,2,2]}}
\end{aligned}
$$

b) Kurveintegral uten bruk av Stokes teorem:

$$
\begin{aligned}
C: \ \vec{r}(t) &= [2\cos t, 2\sin t, 0] \qquad t \in [0, 2\pi] \\
d\vec{r}(t) &= [-2\sin t, 2\cos t, 0]dt \\
\vec{F}(\vec{r}(t)) &= [0 - 2\sin t, 2\cos t, -2\cos t] = [-2\sin t, 2\cos t, -2\cos t] \\
\vec{F} \cdot d\vec{r} &= \vec{F}(\vec{r}(t)) \cdot d\vec{r}(t) = [-2\sin t, 2\cos t, -2\cos t] \cdot [-2\sin t, 2\cos t, 0]dt \\
&= (-2\sin t)(-2\sin t + (2\cos t)(2\cos t) + (-2\cos t) \cdot 0 \\
&= 4\sin^2 t + 4\cos^2 t = 4(\sin^2 t + \cos^2 t) = 4 \cdot 1 = \underline{4}
\end{aligned}
$$

$$\oint_C \vec{F} \cdot d\vec{r} = 4\int_0^{2\pi} dt = 4 \cdot [t]_0^{2\pi} = 4 \cdot (2\pi - 0) = \underline{\underline{8\pi}}$$

c) Kurveintegral vha Stokes teorem ved integrasjon over halvkuleflaten:

$$f(x,y,z) = x^2 + y^2 + z^2 - 4$$
$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right] = [2x, 2y, 2z] = 2[x, y, z]$$
$$|\nabla f| = \sqrt{(2x)^2 + (2y)^2 + (2z)^2} = \sqrt{4x^2 + 4y^2 + 4z^2} = \sqrt{4(x^2 + y^2 + z^2)} = \sqrt{4 \cdot 4} = \sqrt{16} = 4$$
$$\vec{n} = \frac{\nabla f}{|\nabla f|} = \frac{2[x, y, z]}{4} = \frac{1}{2}[x, y, z]$$

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot \vec{n} dS = \iint_R (\nabla \times \vec{F}) \cdot \vec{n} \frac{|\nabla f|}{|\nabla f \cdot \vec{p}|} dA = \iint_R (\nabla \times \vec{F}) \cdot \vec{n} \frac{|\nabla f|}{|\nabla f \cdot \vec{k}|} dA$$
$$= \iint_R [0,2,2] \cdot \frac{1}{2}[x, y, z] \frac{4}{|2[x, y, z] \cdot [0,0,1]|} dA = \iint_R (y + z) \frac{4}{|2z|} dA = \iint_R z \frac{4}{2z} dA = 2\iint_R dA = 2\pi \cdot 2^2 = \underline{\underline{8\pi}}$$

Den delen av integralet som inneholder *y*-delen blir av symmetrigrunner lik null.

d) Kurveintegral vha Stokes teorem ved integrasjon over sirkelskiven i *xy*-planet:

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_R (\nabla \times \vec{F}) \cdot \vec{n} dA = \iint_R (\nabla \times \vec{F}) \cdot \vec{k} dA = \iint_R [0,2,2] \cdot [0,0,1] dA = \iint_R 2 dA = 2\iint_R dA = 2\pi \cdot 2^2 = \underline{\underline{8\pi}}$$

e) Fluksen ut av legemet T:

$$\oiint_{S+R} \vec{F} \cdot \vec{n} dS = \iiint_T \nabla \cdot \vec{F} dV = \iiint_T 0 dV = \underline{0}$$

Fluksen ut av sirkelskiven *R*:

$$\iint_R \vec{F} \cdot \vec{n} dA = \iint_R [z - y, x, -x] \cdot [0,0,-1] dA = \iint_R x dA = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=2} r \cos\theta r dr d\theta = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=2} r^2 \cos\theta dr d\theta$$
$$= \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=2} r^2 \cos\theta dr d\theta = \int_{\theta=0}^{\theta=2\pi} \left[\frac{1}{3} r^3\right]_{r=0}^{r=2} \cos d\theta = \frac{8}{3} \int_{\theta=0}^{\theta=2\pi} \cos d\theta = \frac{8}{3} [\sin\theta]_{\theta=0}^{\theta=2\pi}$$
$$= \frac{8}{3} \cdot (\sin 2\pi - \sin 0) = \underline{0}$$

Fluksen ut av halvkuleflaten *S*:

$$\oiint_{S+R} \vec{F} \cdot \vec{n} dS = \iint_S \vec{F} \cdot \vec{n} dS + \iint_R \vec{F} \cdot \vec{n} dA$$
$$\iint_S \vec{F} \cdot \vec{n} dS = \oiint_{S+R} \vec{F} \cdot \vec{n} dS - \iint_R \vec{F} \cdot \vec{n} dA = 0 - 0 = \underline{\underline{0}}$$

eller direkte integrasjon over halvkuleflaten:

$$f(x, y, z) = x^2 + y^2 + z^2 - 4$$
$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right] = [2x, 2y, 2z] = 2[x, y, z]$$
$$|\nabla f| = \sqrt{(2x)^2 + (2y)^2 + (2z)^2} = \sqrt{4x^2 + 4y^2 + 4z^2} = \sqrt{4(x^2 + y^2 + z^2)} = \sqrt{4 \cdot 4} = \sqrt{16} = 4$$
$$\vec{n} = \frac{\nabla f}{|\nabla f|} = \frac{2[x, y, z]}{4} = \frac{1}{2}[x, y, z]$$

$$\iint_S \vec{F} \cdot \vec{n} dS = \iint_R \vec{F} \cdot \vec{n} \frac{|\nabla f|}{|\nabla f \cdot \vec{p}|} dA = \iint_R [z - y, x, -x] \cdot \frac{1}{2}[x, y, z] \frac{4}{|2[x, y, z] \cdot [0,0,1]|} dA$$
$$= \iint_R [z - y, x, -x] \cdot \frac{1}{2}[x, y, z] \frac{4}{|2z|} dA$$
$$= \iint_R \left((z - y)x + xy - xz\right) \frac{4}{4z} dA = \iint_R 0 \cdot \frac{1}{z} dA = \underline{\underline{0}}$$

f)

$$\nabla\cdot(\nabla\times\vec{F}) = \left[\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right]\cdot\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ F_1 & F_2 & F_3\end{vmatrix}$$

$$= \left[\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right]\cdot\left[\frac{\partial F_3}{\partial y}-\frac{\partial F_2}{\partial z},\frac{\partial F_1}{\partial z}-\frac{\partial F_3}{\partial x},\frac{\partial F_2}{\partial x}-\frac{\partial F_1}{\partial y}\right]$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial F_3}{\partial y}-\frac{\partial F_2}{\partial z}\right)+\frac{\partial}{\partial y}\left(\frac{\partial F_1}{\partial z}-\frac{\partial F_3}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{\partial F_2}{\partial x}-\frac{\partial F_1}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial F_3}{\partial y}\right)-\frac{\partial}{\partial x}\left(\frac{\partial F_2}{\partial z}\right)+\frac{\partial}{\partial y}\left(\frac{\partial F_1}{\partial z}\right)-\frac{\partial}{\partial y}\left(\frac{\partial F_3}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{\partial F_2}{\partial x}\right)-\frac{\partial}{\partial z}\left(\frac{\partial F_1}{\partial y}\right)$$

$$= \frac{\partial^2 F_3}{\partial x\partial y}-\frac{\partial^2 F_2}{\partial x\partial z}+\frac{\partial^2 F_1}{\partial y\partial z}-\frac{\partial^2 F_3}{\partial y\partial x}+\frac{\partial^2 F_2}{\partial z\partial x}-\frac{\partial^2 F_1}{\partial z\partial y}$$

$$= \frac{\partial^2 F_3}{\partial x\partial y}-\frac{\partial^2 F_3}{\partial y\partial x}+\frac{\partial^2 F_1}{\partial y\partial z}-\frac{\partial^2 F_1}{\partial z\partial y}+\frac{\partial^2 F_2}{\partial z\partial x}-\frac{\partial^2 F_2}{\partial x\partial z}$$

$$= \frac{\partial^2 F_3}{\partial x\partial y}-\frac{\partial^2 F_3}{\partial x\partial y}+\frac{\partial^2 F_1}{\partial y\partial z}-\frac{\partial^2 F_1}{\partial y\partial z}+\frac{\partial^2 F_2}{\partial z\partial x}-\frac{\partial^2 F_2}{\partial z\partial x}$$

$$= \underline{\underline{0}}$$

$$\nabla\times(\nabla f) = \nabla\times\left[\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right]f$$

$$= \nabla\times\left[\frac{\partial f}{\partial x},\frac{\partial f}{\partial y},\frac{\partial f}{\partial z}\right]$$

$$= \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z}\end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial z}\right)-\frac{\partial}{\partial z}\left(\frac{\partial f}{\partial y}\right),\frac{\partial}{\partial z}\left(\frac{\partial f}{\partial x}\right)-\frac{\partial}{\partial z}\left(\frac{\partial f}{\partial z}\right),\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)-\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)\right]$$

$$= \left[\frac{\partial^2 f}{\partial y\partial z}-\frac{\partial^2 f}{\partial z\partial y},\frac{\partial^2 f}{\partial z\partial x}-\frac{\partial^2 f}{\partial x\partial z},\frac{\partial^2 f}{\partial x\partial y}-\frac{\partial^2 f}{\partial y\partial x}\right]$$

$$= \left[\frac{\partial^2 f}{\partial y\partial z}-\frac{\partial^2 f}{\partial y\partial z},\frac{\partial^2 f}{\partial z\partial x}-\frac{\partial^2 f}{\partial z\partial x},\frac{\partial^2 f}{\partial x\partial y}-\frac{\partial^2 f}{\partial x\partial y}\right]$$

$$= [0,0,0]$$

$$= \underline{\underline{\vec{0}}}$$