Fourier

Fourier-rekke (Periode 2L)	$f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left[a_n\cos\frac{n\pi t}{L}+b_n\sin\frac{n\pi t}{L}\right]$ $a_n=\frac{1}{L}\int_{-L}^{L}f(t)\cos\frac{n\pi t}{L}dt \quad n=0,1,2,3,\ldots$ $b_n=\frac{1}{L}\int_{-L}^{L}f(t)\sin\frac{n\pi t}{L}dt \quad n=1,2,3,\ldots$	**Alternativt** $a_n=\frac{1}{L}\int_{c}^{c+2L}f(t)\cos\frac{n\pi t}{L}dt \quad n=0,1,2,3,\ldots$ $b_n=\frac{1}{L}\int_{c}^{c+2L}f(t)\sin\frac{n\pi t}{L}dt \quad n=1,2,3,\ldots$
Even funksjon f(-t) = f(t)	$a_n=\frac{2}{L}\int_{0}^{L}f(t)\cos\frac{n\pi t}{L}dt \quad n=0,1,2,3,\ldots$ $b_n=0 \quad n=1,2,3,\ldots$	
Odd funksjon f(-t) = -f(t)	$a_n=0 \quad n=0,1,2,3,\ldots$ $b_n=\frac{2}{L}\int_{0}^{L}f(t)\sin\frac{n\pi t}{L}dt \quad n=1,2,3,\ldots$	

Differensial-ligninger

Svinge-ligning	$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=F$	$mx_{tt}+cx_t+kx=F$
Bølge-ligning	$\frac{\partial^2 y}{\partial t^2}=v^2\frac{\partial^2 y}{\partial x^2}$	$y_{tt}=v^2 y_{xx}$
Varme-ligning	$\frac{\partial u}{\partial t}=\kappa\nabla^2 u$	$u_t=\kappa y_{xx}$

Svinge-ligning m / initialbetingelser

Kloss + fjær uten dempning	$mx''+kx=F(t)$	$0\le t\le L$ $x(0)=0$ $x(L)=0$
	$x(t)=x_0(t)+x_p(t)=c_1\cos(\omega_0 t)+c_2\sin(\omega_0 t)+x_p$ $F(t)=\sum_{n=1}^{\infty} b_n \sin\frac{n\pi t}{L}$ $b_n=\frac{2}{L}\int_0^L F(t)\sin\frac{n\pi t}{L}\,dt$ $x(t)=\sum_{n=1}^{\infty} B_n \sin\frac{n\pi t}{L}$ $=\sum_{n=1}^{\infty}\frac{1}{k-m\frac{n^2\pi^2}{L^2}} b_n \sin\frac{n\pi t}{L}$	$\omega_0=\sqrt{\frac{k}{m}}$
Kloss + fjær med dempning	$mx''+cx'+kx=F(t)$	$0\le t\le L$ $x(0)=0$ $x(L)=0$
	$x_p(t)=\frac{F_0}{\sqrt{(k-m\omega^2)^2+(c\omega)^2}}\sin(\omega t-\alpha)$ $\alpha=\tan^{-1}\frac{c\omega}{k-m\omega^2}$	$F(t)=F_0\sin(\omega t)$
	$F(t)=\sum_{n=1}^{\infty} b_n \sin\frac{n\pi t}{L}$ $b_n=\frac{2}{L}\int_0^L F(t)\sin\frac{n\pi t}{L}\,dt$ $x(t)=\sum_{n=1}^{\infty}\frac{b_n\sin(\omega_n t-\alpha_n)}{(k-m\omega_n{}^2)^2+(c\omega_n)^2}$	

Varme- og bølge-ligning m / initialbetingelser

Varme-ligning m / initialbetingelser **Fourier sinus-rekke**	$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \qquad 0 < x < L$	$u(0,t)=0$ $u(L,t)=0$ $u(x,0)=f(x)$
	$u(x,t)=\sum_{n=1}^{\infty} b_n e^{-\frac{n^2\pi^2 k}{L^2}t}\sin\frac{n\pi x}{L}$ $b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$	
Bølge-ligning m / initialbetingelser **Fourier sinus-rekke**	$\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2} \qquad 0 < x < L$	$y(0,t)=0$ $y(L,t)=0$ $y(x,0)=f(x)$ $y_t(x,0)=g(x)$
	$y(x,t)=\sum_{n=1}^{\infty} A_n \cos\frac{n\pi at}{L}\sin\frac{n\pi x}{L}$ $A_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$	**Problem A** $y(0,t)=0$ $y(L,t)=0$ $y(x,0)=f(x)$ $y_t(x,0)=0$
	$y(x,t)=\sum_{n=1}^{\infty} B_n \sin\frac{n\pi at}{L}\sin\frac{n\pi x}{L}$ $B_n = \frac{2}{n\pi a}\int_0^L g(x)\sin\frac{n\pi x}{L}dx$	**Problem B** $y(0,t)=0$ $y(L,t)=0$ $y(x,0)=0$ $y_t(x,0)=g(x)$
	$y(x,t)=y_A(x,t)+y_B(x,t)$	