Kurve-integral

Kurve-integral	$\int_C f(x,y,z)ds = \int_a^b f(g(t),h(t),k(t))\lvert\vec{r}\,'(t)\rvert dt$	**Kurve-lengde** $\int_C ds = \int_a^b \lvert\vec{r}\,'(t)\rvert dt$
Arbeid	$W = \int_C \vec{F}\cdot\vec{T}ds = \int_C \vec{F}\cdot d\vec{r} = \int_C F_1dx + F_2dy + F_3dz$	
Strømning **Sirkulasjon**	$\int_C \vec{F}\cdot\vec{T}ds = \int_C \vec{F}\cdot d\vec{r} = \int_C F_1dx + F_2dy$ $\oint_C \vec{F}\cdot\vec{T}ds = \oint_C \vec{F}\cdot d\vec{r} = \oint_C F_1dx + F_2dy$	I planet
Fluks	$\int_C \vec{F}\cdot\vec{n}ds = \int_C F_1dy - F_2dx \qquad \vec{n} = \vec{T}\times\vec{k}$	I planet
Fundamental-teoremet for kurve-integraler	$\vec{F}$ vektorfelt med kontinuerlige komponenter over et åpent område D $\Downarrow$ $\exists$ differensierbar funksjon f : $\quad \vec{F} = \nabla f$ $\int_A^B \vec{F}\cdot d\vec{r} = f(B) - f(A)$	
Eksakt differensialform	$F_1dx + F_2dy + F_3dz$ er en differensialform En differensialform er eksakt hvis $Mdx + Ndy + Pdz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$ Differensialform $F_1dx + F_2dy + F_3dz$ er eksakt $\Updownarrow$ $\frac{\partial F_3}{\partial y} = \frac{\partial F_2}{\partial z} \qquad \frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x} \qquad \frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$	
F konservativ (vei-uavhengig)	$\int_C \vec{F}\cdot d\vec{r} = f(b) - f(a)$ $\oint_C \vec{F}\cdot d\vec{r} = 0$ $\vec{F} = \nabla f$ $\text{curl}\vec{F} = \nabla\times\vec{F} = \vec{0}$ $\frac{\partial F_3}{\partial y} = \frac{\partial F_2}{\partial z} \qquad \frac{\partial F_1}{\partial z} = \frac{\partial F_3}{\partial x} \qquad \frac{\partial F_2}{\partial x} = \frac{\partial F_1}{\partial y}$	$\vec{F}$ Konservativ $F_1dx + F_2dy + F_3dz$ Eksakt

Flate-integral

Flate-integral	$\iint\limits_S g\,dS = \iint\limits_R g(x,y,z)\dfrac{\lvert\nabla f\rvert}{\lvert\nabla f\cdot\vec{p}\rvert}\,dA$	**Flate-areal** $\iint\limits_S dS = \iint\limits_R \dfrac{\lvert\nabla f\rvert}{\lvert\nabla f\cdot\vec{p}\rvert}\,dA$
Parameterisert flate-integral	$\iint\limits_S G(x,y,z)\,dS = \iint\limits_R G(f(u,v),g(u,v),h(u,v))\lvert\vec{r}_u\times\vec{r}_v\rvert\,du\,dv$	**Flate-areal** $\iint\limits_S dS = \iint\limits_R \lvert\vec{r}_u\times\vec{r}_v\rvert\,du\,dv$
Fluks	$\iint\limits_S \vec{F}\cdot\vec{n}\,dS$	
Enhetsnormalvektor	$\vec{n} = \dfrac{\nabla f}{\lvert\nabla f\rvert}$ ***f* skalarfunksjon som har flaten *S* som en nivåflate**	$\vec{n} = \dfrac{\vec{r}_u\times\vec{r}_v}{\lvert\vec{r}_u\times\vec{r}_v\rvert}$ **Parameterisert flate**

Green's teorem - Stoke's teorem

2D		**3D**	
Green - Normalform $\text{Divergens} = \text{Flukstetthet} = \frac{\text{Fluks}}{\text{Areal}}$	$\oint\limits_C \vec{F}\cdot\vec{n}\,ds = \oint\limits_C F_1dy - F_2dx$ $= \iint\limits_R \left[\dfrac{\partial F_1}{\partial x}+\dfrac{\partial F_2}{\partial y}\right]dxdy$ $= \iint\limits_R \mathrm{div}\vec{F}\,dA$ $= \iint\limits_R \nabla\cdot\vec{F}\,dA$	$\oiint\limits_S \vec{F}\cdot\vec{n}\,dS = \oiint\limits_S F_1dydz + F_2dzdx + F_3dxdy$ $= \iiint\limits_D \left[\dfrac{\partial F_1}{\partial x}+\dfrac{\partial F_2}{\partial y}+\dfrac{\partial F_3}{\partial z}+\right]dxdydz$ $= \iiint\limits_D \mathrm{div}\vec{F}\,dV$ $= \iiint\limits_D \nabla\cdot\vec{F}\,dV$	**Gauss Divergens**
Green - Tangensialform $Curl = \text{Sirkulasjonstetthet} = \frac{\text{Sirkulasjon}}{\text{Areal}}$	$\oint\limits_C \vec{F}\cdot\vec{T}\,ds = \oint\limits_C \vec{F}\cdot d\vec{r}$ $= \oint\limits_C F_1dx + F_2dy$ $= \iint\limits_R \left[\dfrac{\partial F_2}{\partial x}-\dfrac{\partial F_1}{\partial y}\right]dxdy$ $= \iint\limits_R \mathrm{curl}\vec{F}\cdot\vec{k}\,dA$ $= \iint\limits_R (\nabla\times\vec{F})\cdot\vec{k}\,dA$	$\oint\limits_C \vec{F}\cdot\vec{T}\,ds = \oint\limits_C \vec{F}\cdot d\vec{r}$ $= \oint\limits_C F_1dx + F_2dy + F_3dz$ $= \iint\limits_S \left(\dfrac{\partial F_3}{\partial y}-\dfrac{\partial F_2}{\partial z}\right)dydz + \left(\dfrac{\partial F_1}{\partial z}-\dfrac{\partial F_3}{\partial x}\right)dzdx + \left(\dfrac{\partial F_2}{\partial x}-\dfrac{\partial F_1}{\partial y}\right)dxdy$ $= \iint\limits_S \mathrm{curl}\vec{F}\cdot\vec{n}\,dS$ $= \iint\limits_S \nabla\times\vec{F}\cdot\vec{n}\,d\sigma$	**Stoke**

Green – Gauss – Stokes

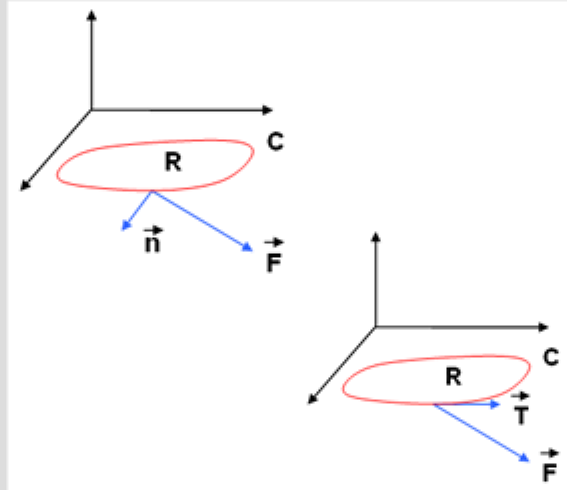
2D

Green - Divergens

$$\Phi = \oint_C \vec{F} \cdot \vec{n}\, ds = \iint_R \nabla \cdot \vec{F}\, dA$$

Green - Curl

$$C = \oint_C \vec{F} \cdot \vec{T}\, ds = \iint_R (\nabla \times \vec{F}) \cdot \vec{k}\, dA$$



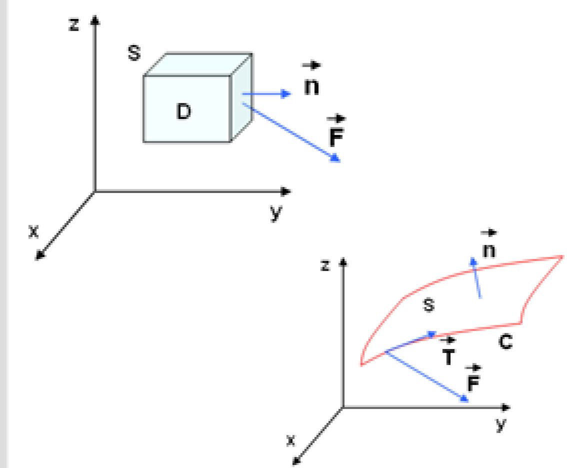
3D

Gauss - Divergens

$$\Phi = \oiint_S \vec{F} \cdot \vec{n}\, dS = \iiint_D \nabla \cdot \vec{F}\, dV$$

Stokes - Curl

$$C = \oint_C \vec{F} \cdot \vec{T}\, ds = \iint_S (\nabla \times \vec{F}) \cdot \vec{n}\, dS$$



Substitusjon i multiple integraler

2D	$\iint_D F(x,y)\,dxdy = \iint_D F(x(u,v),y(u,v))\lvert J(u,v)\rvert\,dudv$ $J(u,v)=\begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$ $\quad J^{-1}(x,y)=\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}$ $\quad J(u,v)J^{-1}(x,y)=1$
3D	$\iiint_D F(x,y,z)\,dxdydz = \iiint_G F(x(u,v,w),y(u,v,w),z(u,v,w))\lvert J(u,v,w)\rvert\,dudvdw$ $J(u,v,w)=\begin{vmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{vmatrix}$ $\quad J^{-1}(x,y,z)=\begin{vmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ w_x & w_y & w_z \end{vmatrix}$ $\quad J(u,v,w)J^{-1}(x,y,z)=1$

Greens form av areal

Tangensiell form	$A=\iint dA=\oint x\,dy$ $\quad \oint F_1dx+F_2dy=\iint\left[\frac{\partial F_2}{\partial x}-\frac{\partial F_1}{\partial y}\right]dA$ $\quad F_2=x \quad F_1=0$ $A=\iint dA=-\oint y\,dx$ $\quad \oint F_1dx+F_2dy=\iint\left[\frac{\partial F_2}{\partial x}-\frac{\partial F_1}{\partial y}\right]dA$ $\quad F_2=0 \quad F_1=-y$
Normalform	$A=\iint dA=\frac{1}{2}\oint x\,dy-y\,dx$ $\quad \oint F_1dy-F_2dx=\iint\left[\frac{\partial F_1}{\partial x}-\frac{\partial F_2}{\partial y}\right]dA$ $\quad F_1=\frac{1}{2}x \quad F_2=\frac{1}{2}y$

Masse - Masse-senter - Treghetsmoment

	Kurve	Flate	Volum
Masse	$M = \int_C \delta ds$	$M = \iint_S \delta dS$	$M = \iiint_V \delta dV$
Masse-senter	$\bar{x} = \frac{1}{M}\int_C x\delta ds$ $\bar{y} = \frac{1}{M}\int_C y\delta ds$ $\bar{z} = \frac{1}{M}\int_C z\delta ds$	$\bar{x} = \frac{1}{M}\iint_S x\delta dS$ $\bar{y} = \frac{1}{M}\iint_S y\delta dS$	$\bar{x} = \frac{1}{M}\iiint_V x\delta dV$ $\bar{y} = \frac{1}{M}\iiint_V y\delta dV$ $\bar{z} = \frac{1}{M}\iiint_V z\delta dV$
Treghetsmoment	$I_x = \int_C (y^2 + z^2)\delta ds$ $I_y = \int_C (x^2 + z^2)\delta ds$ $I_z = \int_C (x^2 + y^2)\delta ds$ $I_L = \int_C r^2 \delta ds$ $R_L = \sqrt{\frac{I_L}{M}}$	$I_x = \iint_S y^2 \delta dS$ $I_y = \iint_S x^2 \delta dS$ $I_L = \iint_S r^2 \delta dS$ $I_o = \iint_S (x^2 + y^2)\delta dS$ $R_L = \sqrt{\frac{I_L}{M}}$	$I_x = \iiint_V (y^2 + z^2)\delta dV$ $I_y = \iiint_V (x^2 + z^2)\delta dV$ $I_z = \iiint_V (x^2 + y^2)\delta dV$ $I_L = \iiint_V r^2 \delta dV$ $R_L = \sqrt{\frac{I_L}{M}}$