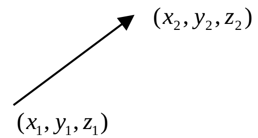
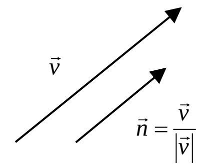
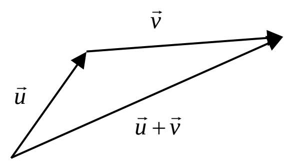
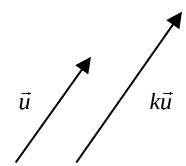
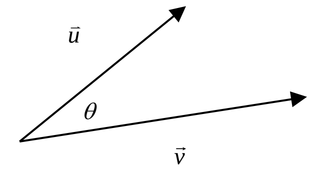
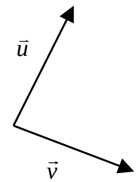
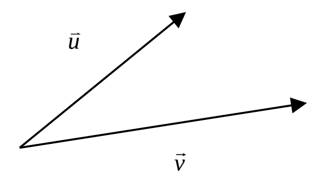
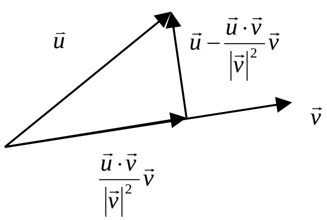
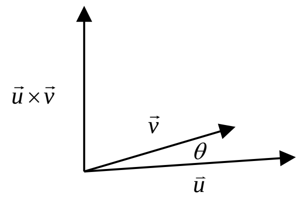
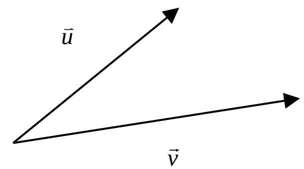
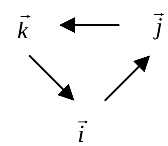
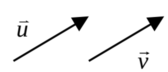
Vektorer og geometri i rommet - Vektor

Vektor	$\vec{v} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$ $= [x_2 - x_1, y_2 - y_1, z_2 - z_1]$ $= [v_1, v_2, v_3]$	
Lengden av en vektor	$\lvert\vec{v}\rvert = \sqrt{v_1^2 + v_2^2 + v_3^2}$	
Enhetsvektor	$\vec{n} = \frac{\vec{v}}{\lvert\vec{v}\rvert}$	
Addisjon	$\vec{u} = [u_1, u_2, u_3]$ $\vec{v} = [v_1, v_2, v_3]$ $\vec{u} + \vec{v} = [u_1 + v_1, u_2 + v_2, u_3 + v_3]$	
Multiplikasjon med skalar	$\vec{u} = [u_1, u_2, u_3]$ $k\vec{u} = [ku_1, ku_2, ku_3]$	
Regneregler for vektorer	$\vec{u} + \vec{v} = \vec{v} + \vec{u}$ $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$ $\vec{u} + \vec{0} = \vec{u}$ $\vec{u} + (-\vec{u}) = \vec{0}$ $0\vec{u} = \vec{0}$ $1\vec{u} = \vec{u}$ $a(b\vec{u}) = (ab)\vec{u}$ $a(\vec{u} + \vec{v}) = a\vec{u} + a\vec{v}$ $(a + b)\vec{u} = a\vec{u} + b\vec{u}$	

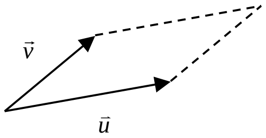
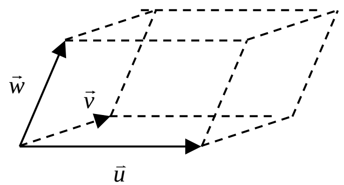
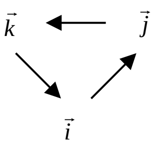
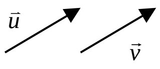
Vektorer og geometri i rommet - Skalarprodukt

Skalarprodukt	$\vec{u}\cdot\vec{v}=\lvert\vec{u}\rvert\lvert\vec{v}\rvert\cos\theta=u_1v_1+u_2v_2+u_3v_3$	
Regler	$\vec{u}\cdot\vec{v}=\vec{v}\cdot\vec{u}$ $(c\vec{u})\cdot\vec{v}=\vec{u}\cdot(c\vec{v})$ $\vec{u}\cdot(\vec{v}+\vec{w})=\vec{u}\cdot\vec{v}+\vec{u}\cdot\vec{w}$ $\vec{u}\cdot\vec{u}=\lvert\vec{u}\rvert^2$ $\vec{0}\cdot\vec{u}=0$	
Ortogonale vektorer	$\vec{u}\perp\vec{v}\quad\Leftrightarrow\quad\vec{u}\cdot\vec{v}=0$	
Projeksjon av u-vektor på v-vektor **Skalarkomponent av u-vektor i retning v-vektor**	$proj_{\vec{v}}\vec{u}=\frac{\vec{u}\cdot\vec{v}}{\lvert\vec{v}\rvert^2}\vec{v}$ $\lvert\vec{u}\rvert\cos\theta=\vec{u}\cdot\frac{\vec{v}}{\lvert\vec{v}\rvert}$	
Vektor skrevet som en sum av ortogonale vektorer	$\vec{u}=\underbrace{\frac{\vec{u}\cdot\vec{v}}{\lvert\vec{v}\rvert^2}\vec{v}}_{\text{Parallell med }\vec{v}}+\underbrace{\left(\vec{u}-\frac{\vec{u}\cdot\vec{v}}{\lvert\vec{v}\rvert^2}\vec{v}\right)}_{\text{Normal på }\vec{v}}$	

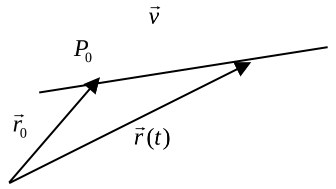
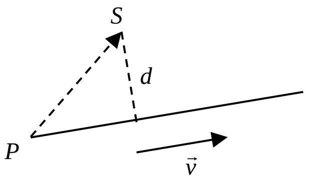
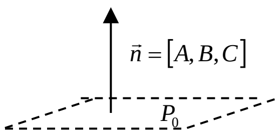
Vektorer og geometri i rommet - Vektorprodukt

Vektorprodukt	$\vec{u}\times\vec{v}=\lvert\vec{u}\rvert\lvert\vec{v}\rvert\sin\theta\,\vec{n}$ $=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ u_1 & u_2 & u_3\\ v_1 & v_2 & v_3\end{vmatrix}$ $=\left[u_2v_3-u_3v_2,\,u_3v_1-u_1v_3,\,u_1v_2-u_2v_1\right]$	
Regler	$(r\vec{u})\times(s\vec{v})=(rs)\vec{u}\times\vec{v}$ $\vec{u}\times(\vec{v}+\vec{w})=\vec{u}\times\vec{v}+\vec{u}\times\vec{w}$ $(\vec{v}+\vec{w})\times\vec{u}=\vec{v}\times\vec{u}+\vec{w}\times\vec{u}$ $\vec{v}\times\vec{u}=-(\vec{u}\times\vec{v})$ $\vec{0}\times\vec{u}=\vec{0}$	
	$\vec{i}\times\vec{j}=\vec{k}$ $\vec{j}\times\vec{k}=\vec{i}$ $\vec{k}\times\vec{i}=\vec{j}$	
Parallelle vektorer	$\vec{u}\parallel\vec{v}\quad\Leftrightarrow\quad\vec{u}\times\vec{v}=\vec{0}$	

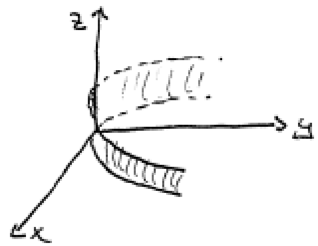
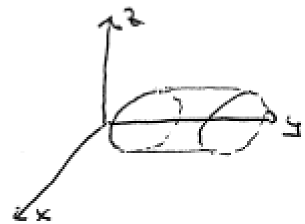
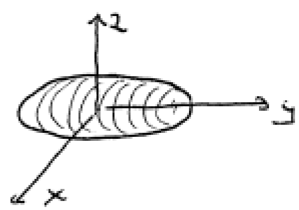
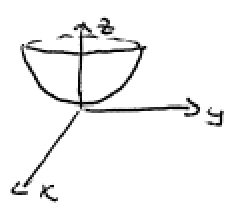
Vektorer og geometri i rommet - Areal - Volum

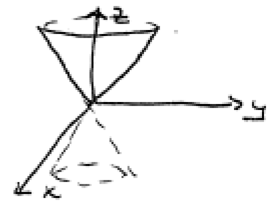
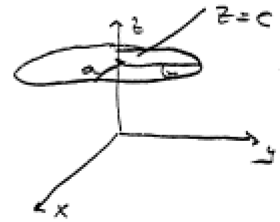
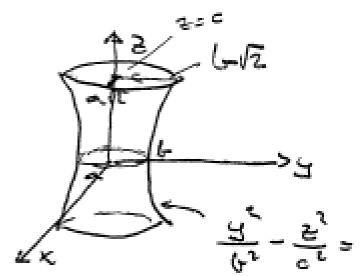
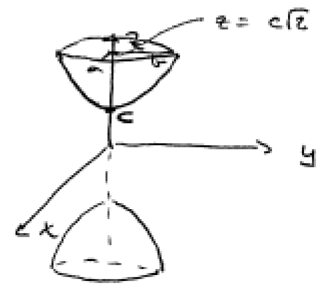
<table>
<tr>
<td>Areal</td>
<td>$A = |\vec{u} \times \vec{v}|$</td>
<td>
</td>
</tr>
<tr>
<td rowspan="2">Volum</td>
<td>$$V = |(\vec{u} \times \vec{v}) \cdot \vec{w}| = \left\| \begin{matrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{matrix} \right\|$$</td>
<td>
</td>
</tr>
<tr>
<td>$\vec{i} \times \vec{j} = \vec{k}$
$\vec{j} \times \vec{k} = \vec{i}$
$\vec{k} \times \vec{i} = \vec{j}$</td>
<td>
</td>
</tr>
<tr>
<td>Parallelle vektorer</td>
<td>$\vec{u} \parallel \vec{v} \quad \Leftrightarrow \quad \vec{u} \times \vec{v} = \vec{0}$</td>
<td>
</td>
</tr>
</table>

Vektorer og geometri i rommet - Linjer - Plan

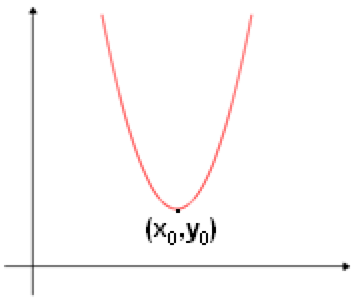
Linje **gjennom P_0** **parallell med v-vektor**	$\vec{r}(t) = \vec{r}_0 + t\vec{v}$ $[x, y, z] = [x_0, y_0, z_0] + t[v_1, v_2, v_3]$ $x = x_0 + tv_1$ $y = y_0 + tv_2$ $x = z_0 + tv_3$	
Avstand **fra et punkt S** **til en linje gjennom P** **parallell med v-vektor**	$d = \dfrac{\lvert \overrightarrow{PS} \times \vec{v} \rvert}{\lvert \vec{v} \rvert}$	
Plan **gjennom P0(x0,y0,z0)** **med normalvektor** $\vec{n} = [A, B, C]$	$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$ $Ax + By + Cz = D \qquad D = Ax_0 + By_0 + Cz_0$	

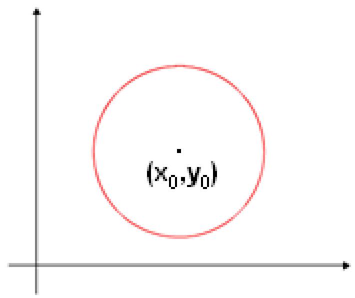
Vektorer og geometri i rommet - Kvadratiske flater

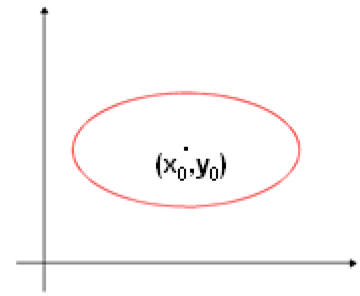
Kvadratiske flater	$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Jz + K = 0$	
Parabolsk sylinder	$y = x^2$	
Elliptisk sylinder	$\frac{x^2}{a^2} + \frac{z^2}{b^2} = 1$	
Ellipsoide	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$	
Elliptisk paraboloide	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$	

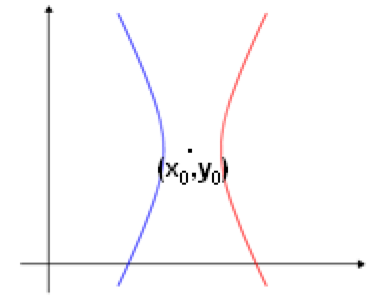
Elliptisk kjegle	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{z^2}{c^2}$	 
Hyperboloide	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ **En flate**	
	$\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ **To flater**	

Parabel – Sirkel – Ellipse – Hyperbel





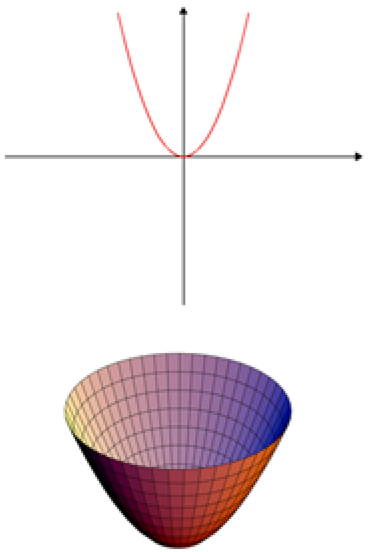




Parabel	Sirkel	Ellipse	Hyperbel
$y = y_0 + k(x - x_0)^2$	$(x - x_0)^2 + (y - y_0)^2 = R^2$	$\left(\frac{x - x_0}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 = 1$	$\left(\frac{x - x_0}{a}\right)^2 - \left(\frac{y - y_0}{b}\right)^2 = 1$
$B^2 - 4AC = 0$	$A = C \qquad B = 0$	$B^2 - 4AC < 0$	$B^2 - 4AC > 0$

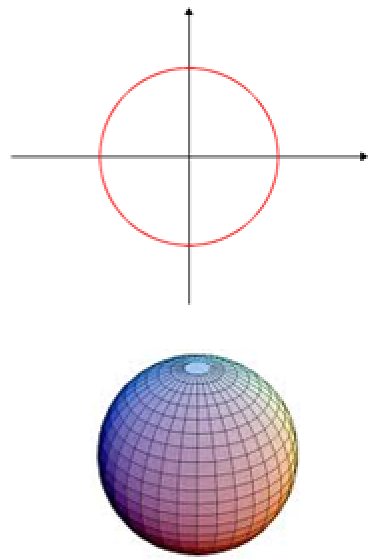
$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

Paraboloide – Kule – Ellipsoide – Hyperboloide



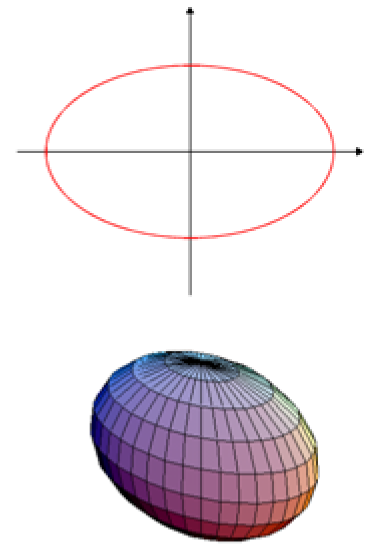
Paraboloide

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-z=0$$



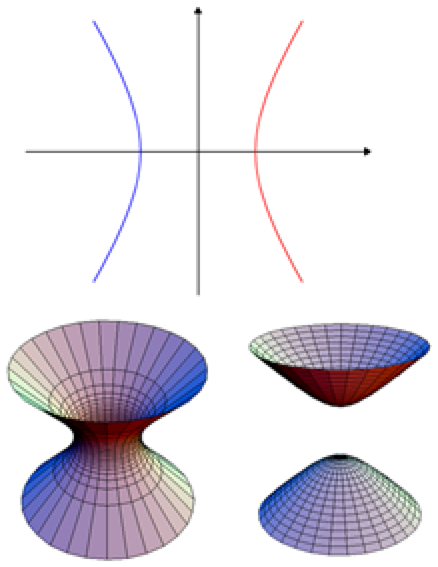
Kule

$$x^2+y^2+z^2=R^2$$



Ellipsoide

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$



Hyperboloide

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=\pm 1$$