

Løsning:

1.

$$\begin{aligned}\bar{B} &= \text{Ingen ener} \\ A \cap B &= \text{Sum øyne oddetall og minst en ener} = \text{Sum øyne oddetall og en ener} \\ A \cup B &= \text{Sum øyne oddetall eller minst en ener} \\ \overline{A \cup B} = \bar{A} \cap \bar{B} &= \text{Sum øyne partall og ingen ener}\end{aligned}$$

2.

$$\begin{aligned}\text{a) } P(A - B) &= P(A) - P(A \cap B) = 0.4 - 0.2 = \underline{\underline{0.2}} \\ \text{b) } P(B - A) &= P(B) - P(B \cap A) = P(B) - P(A \cap B) = 0.5 - 0.2 = \underline{\underline{0.3}} \\ \text{c) } P(\overline{A \cup B}) &= 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)] = 1 - [0.4 + 0.5 - 0.2] = \underline{\underline{0.3}}\end{aligned}$$

3.

$$P(Y = y) = \frac{g}{m} = \frac{\binom{M}{y} \binom{N-M}{n-y}}{\binom{N}{n}}$$

a)

$$P(Y = 1) = \frac{g}{m} = \frac{\binom{1}{1} \binom{8-1}{3-1}}{\binom{8}{3}} = \frac{\binom{1}{1} \binom{7}{2}}{\binom{8}{3}} = \frac{1 \cdot 21}{56} = \frac{3}{8} = \underline{\underline{0.375}}$$

b)

$$P(Y = 2) = \frac{g}{m} = \frac{\binom{2}{2} \binom{8-2}{3-2}}{\binom{8}{3}} = \frac{\binom{2}{2} \binom{6}{1}}{\binom{8}{3}} = \frac{1 \cdot 6}{56} = \frac{3}{28} = \underline{\underline{0.107}}$$

$$\text{c) } P(E \cup G) = P(E) + P(G) - P(E \cap G) = \frac{3}{8} + \frac{3}{8} - \frac{3}{28} = \frac{9}{14} = \underline{\underline{0.643}}$$

$$\text{d) } P(\overline{E \cup G}) = 1 - P(E \cup G) = 1 - \frac{9}{14} = \frac{5}{14} = \underline{\underline{0.357}}$$

4.

$Y =$ Antall staver som virker

$Y \sim \text{Bin}(n, p) = \text{Bin}(10, 0.80)$

$$\begin{aligned}P(\text{Reaktorkat astrofe}) &= P(Y \leq 4) = \sum_{k=0}^4 \binom{10}{k} (0.80)^k (1 - 0.80)^{10-k} = \sum_{k=0}^4 \binom{10}{k} \cdot 0.80^k \cdot 0.20^{10-k} \\ &= \binom{10}{0} \cdot 0.80^0 \cdot 0.20^{10} + \binom{10}{1} \cdot 0.80^1 \cdot 0.20^9 + \binom{10}{2} \cdot 0.80^2 \cdot 0.20^8 + \binom{10}{3} \cdot 0.80^3 \cdot 0.20^7 + \binom{10}{4} \cdot 0.80^4 \cdot 0.20^6 \\ &= 0.000 + 0.000 + 0.001 + 0.006 = \underline{\underline{0.007}}\end{aligned}$$

$$P(\text{Unngå Reaktorkat astrofe}) = 1 - P(\text{Reaktorkat astrofe}) = 1 - 0.007 = \underline{\underline{0.993}}$$

5. a)

$$\hat{\mu} = \bar{X} = \frac{1}{n} \sum_{i=1}^n X_i \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$\hat{\mu} = \bar{X} = \frac{1}{n} \sum_{i=1}^n X_i = \frac{1}{10} [9.36 + 9.75 + 9.23 + 10.32 + 10.07 + 9.68 + 9.96 + 9.70 + 10.15 + 9.68] = \underline{9.79}$$

$$\hat{\mu} \pm u_{\alpha/2} SD(\hat{\mu}) = \hat{\mu} \pm u_{0.025} \frac{\sigma}{\sqrt{n}} = 9.79 \pm 1.960 \cdot \frac{0.30}{\sqrt{10}} = 9.79 \pm 0.186 = \underline{\underline{[9.604, 9.976]}}$$

b)

$$S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2 = \underline{0.117} \Rightarrow S = \underline{0.34}$$

$$\hat{\mu} \pm t_{\alpha/2, n-1} \hat{SD}(\hat{\mu}) = \hat{\mu} \pm t_{0.025, 9} \frac{S}{\sqrt{n}} = 9.79 \pm 2.262 \cdot \frac{0.34}{\sqrt{10}} = 9.79 \pm 0.243 = \underline{\underline{[9.547, 10.033]}}$$

c)

$$H_0 : \mu = 9.62$$

$$H_1 : \mu > 9.62$$

Signifikan ssannsynli ghet :

$$\text{P\AA} \text{st\AA}r H \text{ hvis } p = P(\bar{X} \geq 9.79 \mid \mu = 9.62) < \alpha = 0.05$$

$$p = P(\bar{X} \geq 9.79 \mid \mu = 9.62) = 1 - P(\bar{X} < 9.79 \mid \mu = 9.62)$$

$$= 1 - G\left(\frac{9.79 - 9.62}{\frac{0.30}{\sqrt{10}}}\right) = 1 - G(1.792) = 1 - 0.9633 = 0.0367 < \alpha = 0.05$$

Konklusjon : P\AA st\AA r H_1 , dvs det nye nylonnsn\AA re har st\AA rre strekkstyrke enn det gamle.

Eller :

Beregning av kritisk verdi :

$$\text{P\AA} \text{st\AA}r H \text{ hvis } \bar{X} > k = 9.62 + u_{0.05} \frac{\sigma}{\sqrt{10}} = 9.62 + 1.645 \cdot \frac{0.30}{\sqrt{10}} = 9.62 + 0.156 = 9.776$$

Konklusjon : P\AA st\AA r H_1 , dvs det nye nylonnsn\AA re har st\AA rre strekkstyrke enn det gamle.